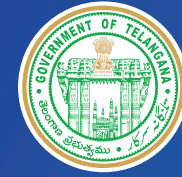




STATE COUNCIL OF EDUCATIONAL RESEARCH & TRAINING  
TELANGANA, HYDERABAD



GOVERNMENT OF TELANGANA  
DEPARTMENT OF SCHOOL EDUCATION



STATE COUNCIL OF EDUCATIONAL RESEARCH & TRAINING  
HYDERABAD

STATE SEMINAR ON MATHEMATICS EDUCATION  
**MATHEMATICS TEACHING  
FOR EMERGING INDIA**



Date: 21st December, 2019

at Godavari Auditorium, SCERT, TS, Hyderabad

**STATE SEMINAR ON MATHEMATICS EDUCATION**

**21<sup>st</sup> DECEMBER, 2019**

**“MATHEMATICS TEACHING FOR EMERGING INDIA”**

Venue : Godavari Auditorium, SCERT, TS, Hyderabad.



**DEPARTMENT OF MATHEMATICS AND SCIENCE**

**STATE COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING,**

**TS, HYDERABAD**

**Smt. B. Seshu Kumari**  
**Director**



**State Council of  
Educational Research  
and Training,  
Opp. L.B. Stadium,  
TS, Hyderabad**

## **FOREWORD**

The State council of Educational Research & Training, Telangana State is the apex organization at the State level, striving for Academic Excellence in the areas of School Education as well as Teacher Education. For the professional growth of the Teachers, the SCERT periodically organizes trainings, symposia, conferences and seminars in different subject areas.

22<sup>nd</sup> December, 1887 is the day on which the great Indian Mathematician, Srinivasa Ramanujan has born. To commemorate the birth of this great genius, the day 22<sup>nd</sup> December announced as “National Mathematics Day”. Every year this day is celebrated by many schools, colleges, other educational institutions and organizations by conducting seminars, elocution etc., on Mathematics. I am glad that on this occasion, the Department of Mathematics and Science of SCERT took initiative to organizing a one-day seminar on the theme “Mathematics Teaching for Emerging India”.

I strongly believe that the deliberations of the seminar will reflect the innovations in Pedagogical practices in Mathematics teaching at different levels i.e., Primary, Upper Primary and secondary levels.

In the light of progressive changes occurring in the curriculum many teachers are implementing innovative practices in their class room teaching. For such teachers it is hoped, this seminar will prove to be a good platform for sharing their successful practices in Mathematics teaching and disseminating their ideas across the teaching community in the state.

**DIRECTOR, SCERT-TS**

*Prof. V. Kannan, [F.A.Sc.](#) F.N.A.*

*Former Pro-Vice Chancellor of University of Hyderabad.*



### *Message*

I am happy to associate myself with the efforts of SCERT to spread the culture and importance of Mathematics among the teachers of that subject (and through them to the general public also). This subject looks awesome for some, because it combines logic, (a new) language and abstract contents simultaneously. But the manner in which it helps one to analytically think, is unparalleled and undoubted.

The seminar on "Mathematics Teaching for Emerging India" gains relevance not only because Mathematics develops the ability of analytical thinking, but also because it has become an indispensable tool for gaining a sound knowledge in other branches of Science as well.

I wish that such seminars be conducted periodically so that more and more sections of the society will be benefited.

Prof. V. Kannan



## **INDEX**

<b>Sl. No</b>	<b>Sub Theme</b>	<b>Title</b>	<b>Name &amp; Address</b>	<b>Page Nos.</b>
1	<b>Challenges in Use of New Technologies in Mathematical Modeling.</b>	Role of technology in teaching – learning Mathematics	Padma. Anil Kumar, SA (PS), ZPHS Boys School, Jogipet, Andole, Sanga Reddy	1-7
2		Introducing ICT in teaching Mathematics with Geogebra	Krishna Korivi, ZPHS Kistapuram, Munugode, Nalgonda	8-12
3	<b>Mathematics for Development of Higher Order Thinking Skills.</b>	Mathematical concepts through games	B. Surya Kumari, TGT Mathematics, Telangana Model School, Nawabpet, Vikarabad.	13-16
4		Mathematics around us	D. Koundinya, SA (PS), Vani Secondary School (Under Teinity Group of Institutions) Centenary Colony, Peddapalli.	17-23
5		Mathematical Teaching for Emerging India	G. Jayasudha, GHM, ZPHS Kamkole, Munipally, Sanga Reddy	24-27
6		Learning of complex judgemental skills such as critical thinking and problem solving	M. Kalpana, SA (Maths), ZPHS Mailardevpally, Ranga Reddy	28-34
7		Vedic approach in teaching Mathematics	M. Muthyam Reddy, Surya's Radiant High School, Siricilla	35-37
8		The effectiveness of an instruction package on the content – specific higher order thinking skills of students.	N. Ravi, SA (Maths), ZPSS Beeravelly, Sarangapur, Nirmal.	38-41
9		Finding the roots of a Quadratic Equation is possible in two methods.	M. Prasanna, GHS, Huzur Nagar, Suryapet	42-45
10		Formula / Generalization for finding the product of 142857 an any number without actual multiplication	Sridhar Kalakonda, SA (Maths), ZPPSS Maripeda, Mahabubabad	46-50

11	<b>Mathematics for Development of Higher Order Thinking Skills.</b>	Hinge questions, Think – Pair - Share	P. Suresh Kumar, GHS, Vijayanagar Colony, Hyderabad	51-56
12		Higher order thinking and 21 <sup>st</sup> century skills	V. Suresh Kumar, PGT (Maths), TSMS Morgi, Nagalgidda, Sanga Reddy	57-59
13		Development of Higher Order Thinking through Open-ended Questions	Dr. Rrambabu Akinapally, Rtd., Associate Professor, GCTE-, Warangal.	60-64
14	<b>Handling Abstractions in Mathematics</b>	Addition and subtraction formula infractions.	Athinarapu Vanaja, SA (Mathematics), MPUPS Thuppareddypally, Vangoor, Nagarkurnool.	65-74
15		Handling abstractions in UP level and Secondary level mathematics by using VEDIC Mathematics Sutras.	M. Srinivas, SA (Maths), ZPHS Dandepally, Elkathurthy, Warangal Urban.	75-80
16		Measuring Golden Ratio by using an instrument.	M. Radhika, CRT (Maths), KGBV, JAM, Sarangapur, Nirmal.	81-84
17		The relation between Mathematical objects and thinking process.	K. Rakesh Kumar, UPS Kachikallum Nellikuduru, Mahabubabad.	85-92
18		Primitive Pythagorean Triples	T. Srinivas, ZPHS Kyasampally, Kamareddy	93-95
19		Effectiveness of using Geogebra software and stimulations on Mathematics Learning	K. Ramaiah, SA, ZPHS Tatikonda, Jangoan and Sardar Dharmendra Singh, SA, ZPHS Mannur, Adilabad	96-105
20		Smart Teacher Learning Resources and Tools	Dr. Goli Srilatha, LB College of Education, Warangal	106-110
21		The effect of teaching Mathematics through flipped Learning strategies	V. Padma Priya, SA (Maths), Govt. Nehru Memorial High School, Malakpet, Hyderabad.	111-118

# **STATE LEVEL MATHEMATICS SEMINAR-2019**

## **PERSONAL DETAILS**

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**MAIN THEME :** MATHEMATICS TEACHING FOR EMERGING INDIA

**SUB THEME :** CHALLENGES IN USE OF NEW TECHNOLOGIES IN MATHS  
MODELLING

**TITLE OF THE TOPIC : ROLE OF TECHNOLOGY IN TEACHING-LEARNING MATHEMATICS**

**OBJECTIVE:-**

Today, in many locations around the world, there is a significant gap between the knowledge and skills students learn in school and the knowledge and skills workers need in workplaces and communities.

Employers report that they need students who are better prepared in skills such as professionalism and work ethic, oral and written communication, teamwork and collaboration, critical thinking and problem solving, application of information technology, and leadership.(Sources: The Partnership for 21st Century Skills, enGauge, and SCANS Report highlight different skills and call them “21<sup>st</sup>Century Skills.”).

So the emphasis in schools is increasingly on *learning how to learn*, rather than just acquiring specific technical skills that keep changing anyway.

Schools today face ever-increasing demands in their attempt to ensure that students are well equipped to enter the workforce and navigate a complex world. Research indicates that computer technology can help support learning, and that it is especially useful in developing the higher order skills of critical thinking, analysis, and scientific inquiry

Mathematics, to most, is a complex and difficult subject. The tendency for most students is to consider the subject as one that is boring, thus, creating lack of interest in the topics being discussed. This poses a great challenge for teachers and educators, especially in the primary and intermediate levels, wherein a good study habit and a firm grasp of basic concepts should be developed

**2.DESIGN OF THE INNOVATION:-**

**WHY TO USE TECHONOLOGY :-**

**2.1 Change of Scenario**

Mathematics is regarded as the queen of all Sciences. For long, the role of Mathematics was limited to purely academic domain. Now, the role of Mathematics is not restricted to purely academic domain. It has entered the domain of Technology and Industry. New fields in Mathematics such as Operation Research,



Control theory, Signal Processing and cryptography have been generated which need technology.

Technology can reduce the effort devoted to tedious computations and increase students' focus on more important mathematics.

## **2.2 Technology focuses Student's thinking**

Technology can be useful to the extent it focuses student thinking in ways that are germane, not extraneous.

In primary school, it is important to learn to do arithmetic fluently. Using technology to do this thinking for the student would be inappropriate. In secondary school, however, students have mastered arithmetic and should be focused on more advanced skills and concepts. Computational support can be very important.

## **2.3 Use of technology makes Ideas Tangible.**

Piaget discovered that children first develop ideas concretely and later progress to abstractions (Piaget, 1970). In designing learning environments, it is often helpful to apply this principle in reverse: to help students learn an abstract idea, provide them with more tangible visualizations.

Researchers have found that when technology makes abstract ideas tangible, teachers can more easily (Bransford, Brown, & Cocking, 1999; Roschelle et al., 2001; diSessa, 2001):

- Build upon students' prior knowledge and skills.
- Emphasize the connections among mathematical concepts.
- Connect abstractions to real-world settings.
-

## WHAT IS TECHNOLOGY:

Technology is the making, modification, usage, and knowledge of tools, machines, techniques, crafts, systems, methods of organization, in order to solve a problem, improve a preexisting solution to a problem, achieve a goal or perform a specific function. It can also refer to the collection of such tools, machinery, modifications, arrangements and procedures

## HOW TO USE TECHNOLOGY AS TOOLS OF TEACHING

There are various types of technologies currently used in traditional classrooms.

Among these are: Radio, television, audio tape, video tape, slide projector, overhead projector are of passive learning when interaction of the learner is less.

**4.1. Computer in the classroom:** Having a computer in the classroom is an asset to any teacher. With a computer in the classroom, teachers are able to demonstrate a new lesson, present new material, illustrate how to use new programs, and show new websites.

**Class blogs and wikis:** There are a variety of Web 2.0 tools that are currently being implemented in the classroom. Blogs allow for students to maintain a running dialogue, such as a journal, thoughts, ideas, and assignments that also provide for student comment and reflection. Wikis are more group focused to allow multiple members of the group to edit a single document and create a truly collaborative and carefully edited finished product.

**Wireless classroom microphones:** Noisy classrooms are a daily occurrence, and with the help of microphones, students are able to hear their teachers more clearly. Children learn better when they hear the teacher clearly.

**Mobile devices:** Mobile devices such as clickers or smart phone can be used to enhance the experience in the classroom by providing the possibility for professors to get feedback.

**Interactive Whiteboards:** An interactive whiteboard that provides touch control of computer applications. These enhance the experience in the classroom by showing anything that can be on a computer screen. This not only aids in visual learning, but it is interactive so the students can draw, write, or manipulate images on the interactive whiteboard

**Digital video-on-demand:** Digital video eliminates the need for in-classroom hardware (players) and allows teachers and students to access video clips immediately by not utilizing the public Internet.

**Online media:** Streamed video websites can be utilized to enhance a classroom lesson.

**Online study tools:** Tools that motivate studying by making studying more fun or individualized for the student.

**Digital Games:** The field of educational games and serious games has been growing significantly over the last few years. The digital games are being provided as tools for the classroom and have a lot of positive feedback including higher motivation for students. There are many other tools being utilized depending on the local school board and funds available. These may include: digital cameras, video cameras, interactive whiteboard tools, document cameras, or LCD projectors.

#### 4.2 Soft ware used for teaching learning Mathematics

- Graphic Calculators
- Dynamic graphing tools (Geo gebra )
- Dynamic geometry tools
- Microsoft Excel / spreadsheet
- Microsoft Mathematics
- Geo Gebra

- Auto shape
- Mat lab

**4.3. Learning resource centre (Indian system of Education)/Websites** Thousands websites provide e-resource for both offline and online teaching –learning.

IGNOU (<http://www.ignou.ac.in/>)

The Indira Gandhi National Open University  
(IGNOU),

[http://www.ncert.nic.in/NCERTS/textbook/textbook.  
htm](http://www.ncert.nic.in/NCERTS/textbook/textbook.htm)

The website is a e-resource for syllabus, online text books, other publications such as sample question papers and multimedia packages which helps both the students and teachers in teaching learning Mathematics.

<http://www.ciet.nic.in>

### **OUT COMES OF THE INNOVATION**

Technology provides New Ways of Learning

New Roles of the Teacher

Teaching Mathematics Better and Teaching Better Mathematics

### **IMPLICATIONS:**

THE STUDENTS WILL LEARN MATHEMATICS VERY JOYFUL AND EASY

TEACHING WILL BECOME VERY FLEXIBLE TO THE TEACHER



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[http://cte.uwaterloo.ca/teaching\\_with\\_technology/](http://cte.uwaterloo.ca/teaching_with_technology/)

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**THANK YOU**

**Write up for the SEMINAR at the SCERT on 21<sup>st</sup> Dec, 2019**

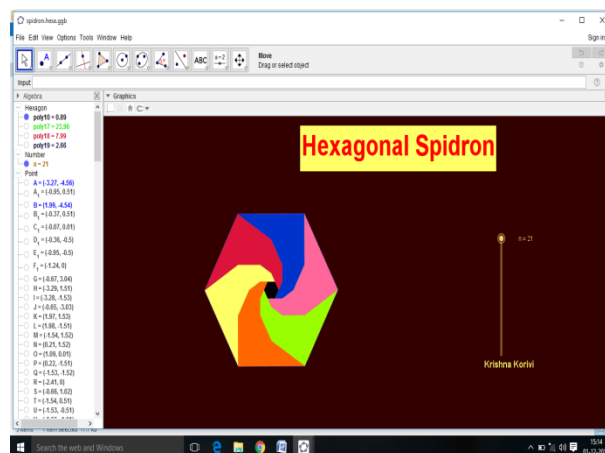
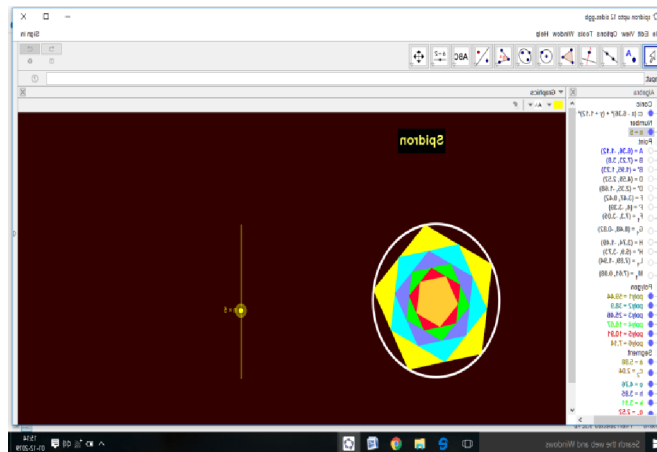
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**Sub Theme:** Challenges in use of New Technologies in Mathematical Modelling

**Title of the Topic:** Introducing ICT in teaching Mathematics with Geogebra

**Objectives:** While teaching Mathematics we are facing so many hurdles and obstacles. Some times its very hard to introduce the concept because some concepts are abstract. We are unable to make them to understand. By using ICT we can make our teaching much effective.

**Design of the Innovation :** The main motto of ICT is Introducing pictorial representations with the help of animations and to easily understand the academic standards like VISUALIZATION AND REPRESENTATION ,Reasoning and Proof, Problem Solving, Connection, Communication. And we can give the thought provoking questions by using ICT.

Now let us discuss the world famous mathematical application GEOGEBRA. It is introduced in 56 languages and more than Ten crores of Teachers and students are using all over the world. The main motto of Geogebra is “GEOGEBRA PROMOTES EQUITY IN MATHEMATICS CLASS ROOM”. There are so many forums are running to promote geogebra and as well as the assistance to teachers and students too.

**Description of the Innovation :** While teaching mathematics to introduce the Concept we use TLM's but we cannot animate them. By using Geogebra we can animate the pictures, graphs , geometrical figures and statistical data. Geogebra is user friendly for the normal persons who doesn't have the soft skills also. Geogebra is used in Calculus, Geometrical concepts , Coordinate geometry, Algebra, Number Theory, Derivations, Physics concepts, Chemistry, Biological sciences, Social studies solar system.

### Topics dealt with Geogebra

**1.AREAS:** The basic figure to find the formulae for the area of any polygon is Rectangle

By transforming polygons as Rectangles we can find the formulae to find the area of them.

We can animate Triangle,Quadrilateral,Trapezium,Parallelogram,Rhombus,Sqaure,Circle,Pentagon,Hexagon as Rectangle and hence we can find the formulae

**2.Angles:**By using sectors We can generalise The sum of the interior and exterior angles of a Triangle, Quadrilateral, Pentagon, Hexagon

And also we can animate all the properties regarding angles of any polygon

**3.Properties:**We can show the properties of tangents to the radius and the length of the tangents from external point to the circle

**4.Spidrons:** Its very hard to show the mathematical spidrons using TLM's.But with geogebra we can

**5.Fundamental Operations :** we can show the Addition, subtraction, Multiplication, Division of integers and fractions

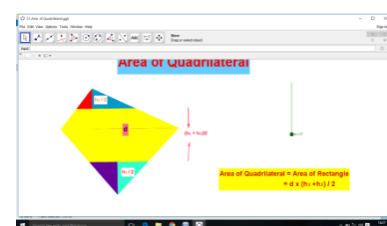
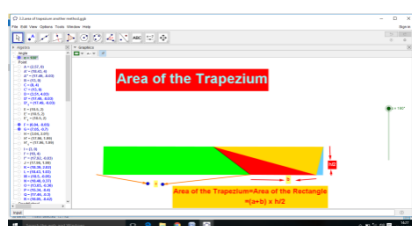
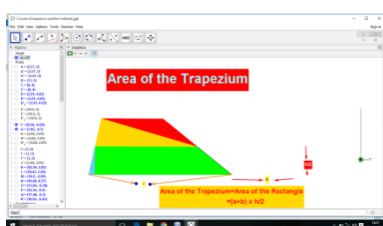
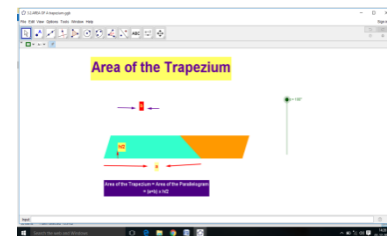
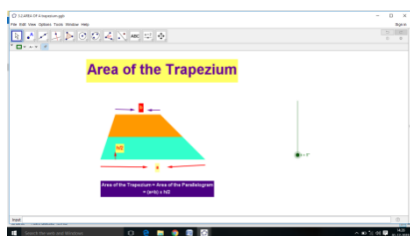
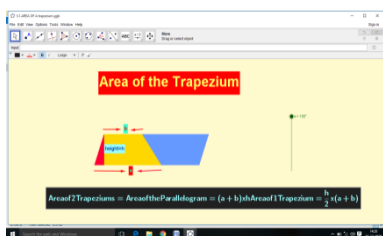
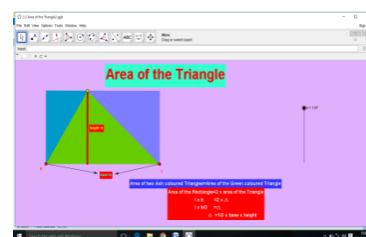
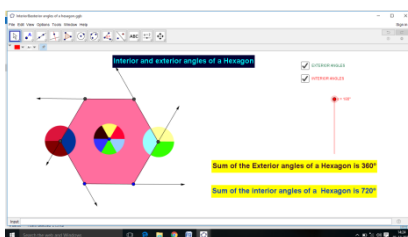
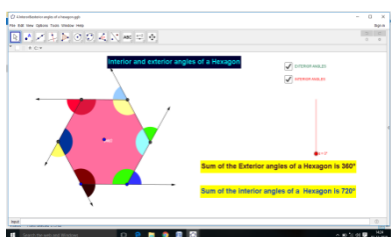
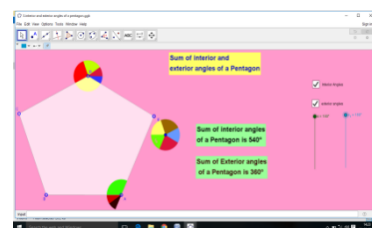
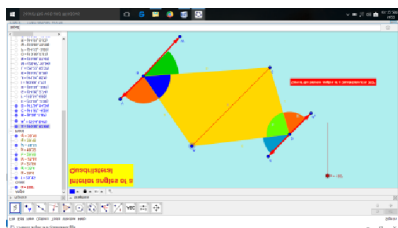
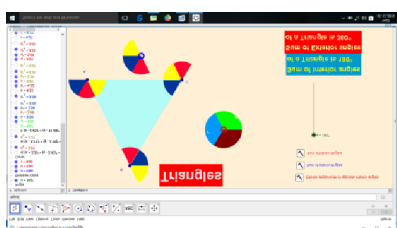
6. Comparing Fractions: Geogebra easily give the comparison between the fractions with Pictorial representation.

7. Trigonometric Ratios: all the trigonometrical operations and the heights and distances chapter we can make easy to understand with the help of geogebra.

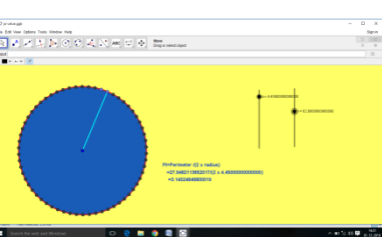
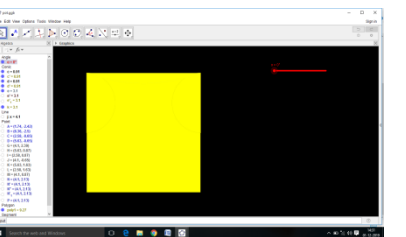
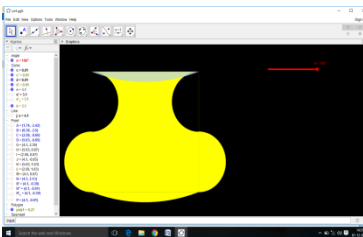
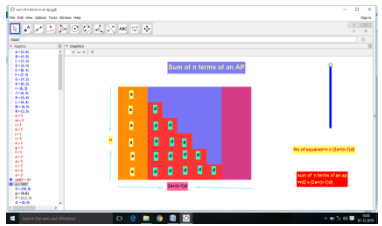
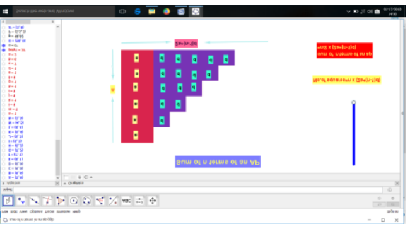
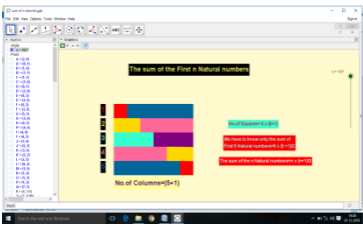
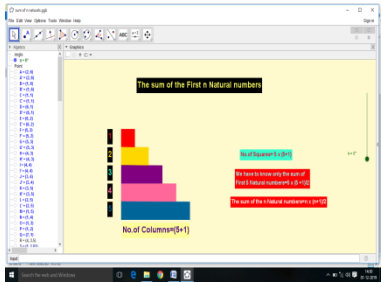
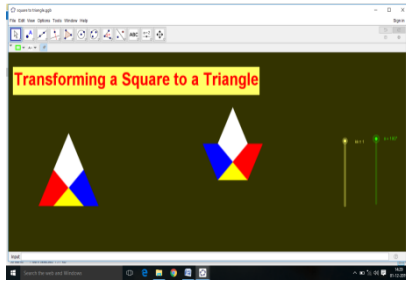
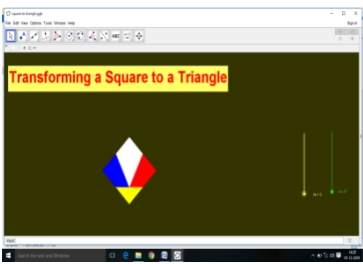
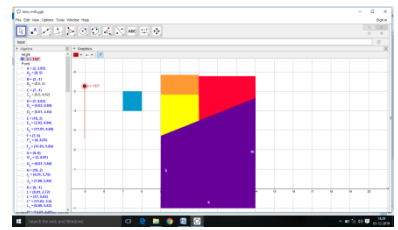
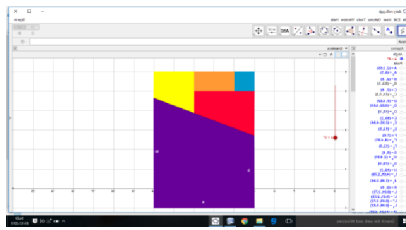
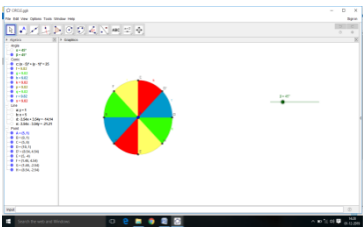
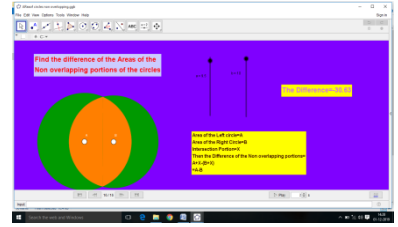
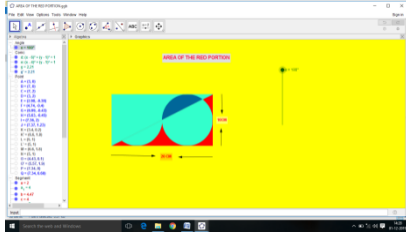
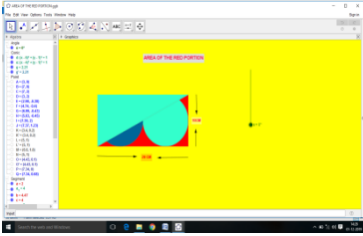
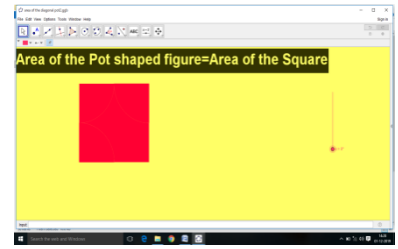
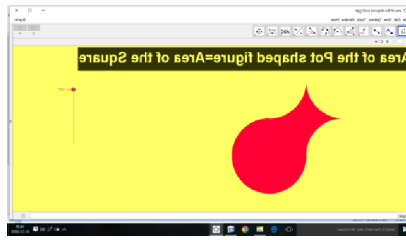
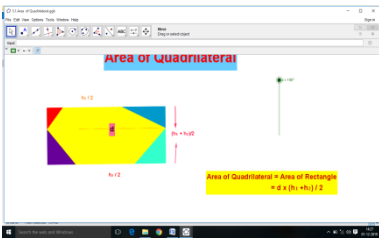
8. Theorems: The thales theorem, Phythagorean theorem, Brahmaguptha theorem We can prove with geogebra.

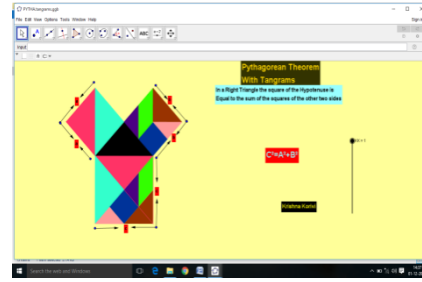
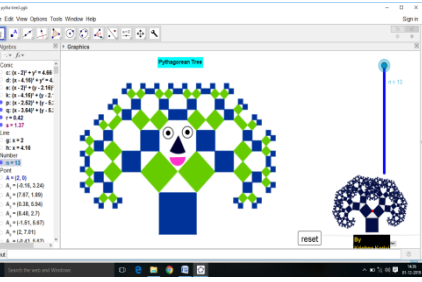
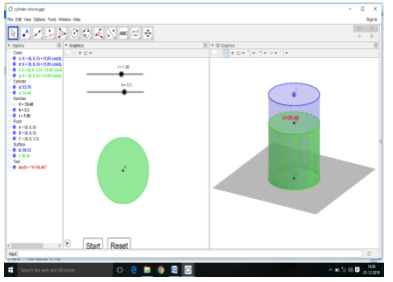
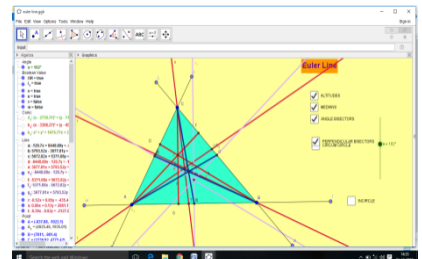
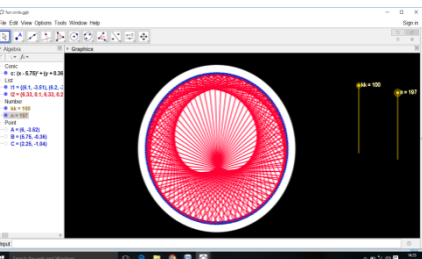
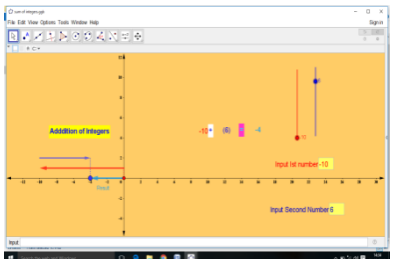
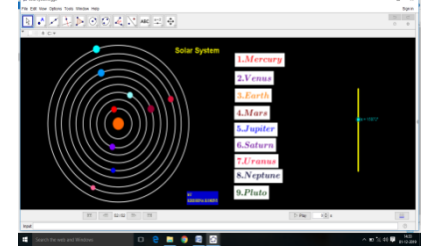
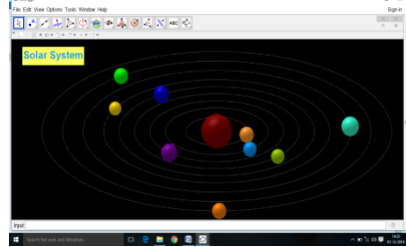
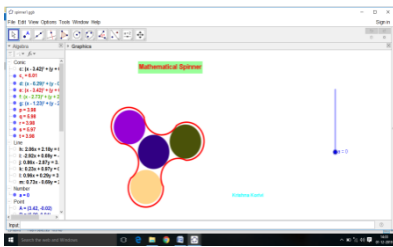
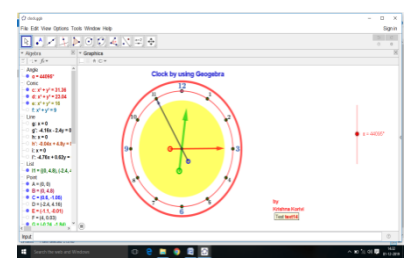
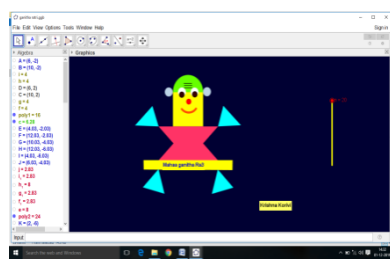
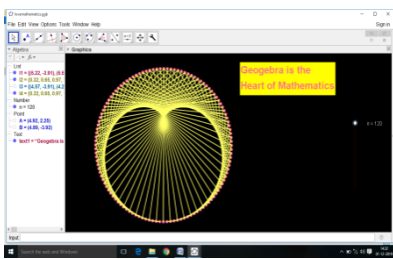
**Out come of the Innovation:** Digital class will attract the students to learn not only the Mathematics but also the sciences too..By using geogebra we can prepare self evaluation exams also..The students will learn the logical reasoning with pictorial representation..

Here we have some of the videos made for the Class room teaching









**Implications :** We can extend the Geogebra not only for mathematics we can extend it to the Social studies Physics and chemistry and biology too.

**References :** Mathematical modelling and [www.geogebra.com](http://www.geogebra.com) in youtube class room resources for geogebra users.

**Note:** I have started A **youtube channel named as Krishna korivi** to introduce geogebra in telugu for the Telugu users of geogebra..and i have uploaded some videos..explaining how to make mathematical modelling videos using GEOGEBRA.

Here it is the youtube link : <https://youtu.be/2aB6jHq1DGg>

## MATHEMATICS TEACHING FOR EMERGING INDIA

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Sub- theme: Mathematics for the development of higher order skills.

Title of the topic: MATHEMATICAL CONCEPTS THROUGH GAMES

### Objectives:

- 1) To make the student learn the concept through a game.
- 2) To maximize the student participation in the activity.
- 3) To develop kinesthetic skills.
- 4) To make the learning realistic and practical.
- 5) To motivate the student learn Mathematics without fear.

### Design of the method:

- a) Level 1 (GAMING PHASE)
  - i) Selection of the concept (topic)
  - ii) Acquiring the necessary material.
  - iii) Preparing the students for the activity.
  - iv) Giving instructions to the students.
  - v) Student participation in the game.
- b) Level 2 (ACTIVITY CUM WRITTEN PHASE)
  - i) Filling activity sheets by students (group or individual)

Description of the model:

LEVEL 1 (GAMING PHASE)

In this phase the student learns the concept through a game.

For example if the concept selected is “ADDITION OF INTEGERS”.

Material required: Face masks with positive + and negative – signs.

Students were divided into 3 groups. One group of students wears positive + masks and the other group wears negative – masks and the third group of students gives them different integers to find their sum.

For instance, a student from third group says “SUM OF POSITIVE 5, POSITIVE 3 AND NEGATIVE 11” ie;  $5+3+(-11)$ .

Now 5 students wearing positive masks, 3 students wearing positive masks and 11 students wearing negative masks stands in two rows (all positives in one row and all negatives in the second row at the back of positives).

+ + + + + + + +  
- - - - - - - - - -

Then the students wearing one positive mask and one negative mask kneels down thus forming a “ZERO PAIR”. When all possible “ZERO PAIRS” are formed the remaining positives or negatives form the answer to the question given by the third group. In the above example, 3 negatives are left after forming “ZERO PAIRS” so the answer is -3.

- - -

The game is repeatedly played with different integers for ample number of times so that the student concludes that whichever integers are more that becomes the sign of the resultant sum.

Generalization: The student generalize that  $a+(-b) = a-b$ . And the sign of the sum will be whichever is maximum (either positive or negative).

LEVEL -2 (ACTIVITY CUM WRITTEN PHASE)

In this level the teacher gives the activity sheets to students with an example and explains it. And then students are asked to complete the activity. Here the student applies what he/she has learnt in level1 to concretize the concept. The activity can be given to students in a group or individual.

## ACTIVITY SHEET

Observe the following example and complete the activity

Example:  $2 + (-15) + 4 - 5 = 2 - 15 + 4 - 5 = -20 + 6 = -14$

POSITIVES	NEGATIVES
2	15
4	5
+6	-20

- 1)  $6 + (-3) + 10$
- 2)  $-12 + (-11) + 5$
- 3)  $23 + (60) + (-35)$

There can be 10 to 15 questions in the activity sheet.

Role of teacher:

The teacher using the gaming method should select the topic which is appropriate for the game. The material needed (if necessary) for the game should be kept ready. The instructions to be given to the students should be sorted out in an order. The teacher gives the step by step instructions to the students and they follow and act accordingly. Here the teacher acts as a guide or the supportive agent and a facilitator and supervises the work.

Outcome:

The outcome of the gaming method always gives positive result and helps in better learning than usual methods. The student ends up in learning the concept in a meaningful way rather than rote memorization. Since the student enjoy and learn in every phase of the game, the concepts are concretized in the initial stages and lowers the abstractness of the concepts and helps in higher order thinking in the later stages.

Advantages of gaming method:

- 1) The student learns the concept thoroughly and remembers for a longer time.
- 2) Student makes the connection between what they know and what they are going to learn.
- 3) The group learning lowers the level of anxiety and stress.
- 4) The student learns the concept with deeper understanding.
- 5) Shows increased individual achievement compared to working in alone.
- 6) The flexibility of the rules makes the method feasible to the teacher.

Disadvantages of gaming method:

- 1) It is a time consuming method and sometimes may not be completed in 45 minutes of a period.
- 2) The method may not be applicable to all concepts.
- 3) It is not applicable for higher class students.

Implications:

- 1) It can be used in teaching fundamentals of Mathematics.
- 2) It can be used as an evaluation tool as well.

Conclusion:

Learning is a continuous process. Teaching and learning Mathematics cannot be improved in isolation. Especially teaching Mathematics to the lower standard and who do not have a strong foundation at their initial levels (primary level) or to the students with learning gap is a tough task. It is challenging for a teacher to shift the mindset of the student from “Fear of Maths” to “Fun out of Maths”.

The so called Fear of Maths hinders the student from effectively taking Mathematical problems. Many people think of Mathematics as an extremely tough subject that they cannot master. This negative attitude stops them from focusing on the subject/ problem which they are tackling. The students are left in isolation and they do not understand the concepts of the higher level if they do not know basics. This hinders their higher order thinking skills in the later stages of learning. Therefore, sound knowledge of concepts at the initial level is much important.

To overcome this phobia/ fear of Mathematics in students, the Mathematics teacher should play a constructive role. The teacher has to lit the fire of curiosity to learn Mathematics through different methods. The gaming method helps in quick learning of concepts and make the learning realistic. Concretizing concepts in primary and upper primary levels helps the student in learning Mathematics in a better way in the higher levels. The method awakes the enthusiasm to learn Mathematics with ease and creates interest towards the subject in future learners as well.

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THANK YOU

**PAPER PRESENTATION FOR THE STATE LEVEL  
MATHEMATICS SEMINAR ON THE OCCASION OF  
NATIONAL MATHEMATICS DAY**

**Presented by**

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**Main Theme**

Mathematics Teaching for Emerging India

**Sub Theme**

Mathematics for Development of Higher Order Thinking Skills

**Organized by**

State Council Of Educational Research And Training,

Department Of Mathematics And Science,

Telangana

# Mathematics Teaching for Emerging India

## Mathematics for Development of Higher Order Thinking Skills

**Abstract**– Mathematics learning has an important role in improving the quality of human resources in order to compete in the era of globalization and modernization. In order to achieve these objectives, mathematics learning should be able to facilitate individuals to develop higher order thinking skills. Efforts have been made to improve Mathematics teaching in schools, yet, students performance is poor in mathematics subject. Hence, The purpose of this paper is to evaluate the traditional methods of teaching and to suggest other useful teaching methods that can be attempted in imparting higher order thinking skills in the students. The use of innovative methods in educational institutions has the potential not only to improve education, but also to empower people, strengthen governance and galvanize the effort to achieve the human development goal for the country.

### I. Introduction

“Higher order thinking is thinking on a level that is higher than memorizing facts or telling something back to someone exactly the way it was told to you”. we do something with the facts. We must understand them, infer from them, connect them to other facts and concepts, categorize them, manipulate them, put them together in new or novel ways, and apply them as we seek new solutions to new problems”.

“higher order thinking occurs when a person takes new information and information stored in memory, interrelates or rearranges and extends this information to achieve a purpose or find possible answers in perplexing situations Despite the challenges in delivering the content of the mathematics syllabus, teachers nowadays are also facing difficulties in catering for a wide range of students’ abilities. Each student has the ability to learn mathematics at a different rate. However, students’ potential for learning mathematics can be lost if it is not discovered and supported at the appropriate time This is because there are students who cannot perceive the importance of being involved in such assessments. Consequently, they do not put much effort into answering the questions they are given. However, the students’ poor achievement in national examinations for primary and secondary school students with the increased emphasis on higher order thinking skills questions serves as solid evidence of students’ lack of mastery in higher-order thinking skills. The weak performances of students in international and local assessments demonstrate that the intended target set by the Ministry of Education has not yet been fully achieved. Hence, the factors that hinder students’ mastery of higher order thinking skills in mathematics should be explored



## II. Objectives

These objectives illustrates practical activities to improve learning and teaching skills. It will support improvement by utilising higher order thinking skills by tackling the following key areas:

- a. problem solving: seeking and identifying strategies and reasoning
- b. comprehension and interpretation of statistics
- c. flexibility of thinking
- d. using and understanding appropriate mathematical vocabulary
- e. identifying the steps and using number of operations
- f. realising the importance of accurate calculations
- g. applying inverse operations.
- h. extending knowledge and understanding of higher order thinking skills
- i. planning learning and teaching
- j. providing strategies to support learning
- k. enabling opportunities for challenge

## III. Methodology

Mathematics is a methodical application of matter. It is so said because the subject makes a man methodical or systematic. Mathematics makes our life orderly and prevents chaos. Certain qualities that are nurtured by mathematics are power of reasoning, creativity, abstract or spatial thinking, critical thinking, problem-solving ability and even effective communication skills.

Mathematics is the cradle of all creations,

without which the world cannot move an inch. Be it a cook or a farmer, a carpenter or a mechanic, a shopkeeper or a doctor, an engineer or a scientist, a musician or a magician, everyone needs mathematics in their day-to-day life. Even insects use mathematics in their everyday life for existence. Snails make their shells, spiders design their webs, and bees build hexagonal combs. There are countless examples of mathematical patterns in nature's fabric. Anyone can be a mathematician if one is given proper guidance and training in the formative period of one's life. A good curriculum of mathematics is helpful in effective teaching and learning of the subject.

Here are a few strategies that can be followed-

- a. Puzzles and Riddles
- b. Inquiring Skills
- c. Conceptualizing Skills
- d. Reasoning Skills

### a. Puzzles and Riddles

**Design of the Innovation:** Experience says learning mathematics can be made easier and enjoyable if our curriculum includes mathematical activities and games. Maths puzzles and riddles encourage and attract an alert and open-minded attitude among youngsters and help them develop clarity in their thinking.

**Description of the Innovation:** Emphasis should be laid on development of clear concept in mathematics in a child, right from the primary classes. If a teacher fails here, then the child will develop a phobia for the

subject as he moves on to the higher classes. For explaining a topic in mathematics, a teacher should take help of pictures, sketches, diagrams and models as far as possible. As it is believed that the process of learning is complete if our sense of hearing is accompanied by our sense of sight. Open-ended questions should be given to the child to answer and he/she should be encouraged to think about the solutions in all possible manners. The child should be appreciated for every correct attempt. And the mistakes must be immediately corrected without any criticism.

**Outcome of the Innovation:** The greatest hurdle in the process of learning mathematics is lack of practice. Students should daily work out at least 10 problems from different areas in order to master the concept and develop speed and accuracy in solving a problem. Learning of multiplication-tables should be encouraged in the lower classes

**Example:**

Split the students in the class to 2 or 3 batches. Ask the students to prepare few puzzles by themselves from the concepts that have been covered so far

Apple, Banana and Cherry Riddle : The numbers  $a$ ,  $b$ ,  $c$  are positive integers. An Apple cost Rs. $a$ , a banana costs Rs. $b$  , a cherry costs Rs. $c$  . The cost of  $b$  apples, ' $b$ ' bananas, and ' $a+b$ ' cherries is Rs.77. What would be the cost for 1 Apple, 2 Bananas and 1 Cherry?

## **b. Inquiring Skills**

**Design of the Innovation:** Inquiring involves discovering or constructing knowledge through questioning or testing a hypothesis. Observation, analysis, summarizing and verification are the essential elements in carrying out inquiring activities.

**Description of the Innovation:** Inquiring activities mainly involve self-learning processes, but suitable guidance from teachers are sometimes necessary depending on the abilities of students and the complexity of the activities. Posing questions is one popularly adopted means to guide students to make exploration. In fact, well-designed questions are useful to stimulate students to discover similarities, differences, patterns and trends. Students may also be asked to test mathematical conjectures, which enable them to participate in a more active role in the learning process.

**Outcome of the Innovation:** Inquiring activities designed should cope with the abilities of students so that they can enjoy the discovery of mathematical results. Moreover, it may be more effective to arrange students in small groups (whenever possible) because it is easier for them to put forward their ideas. The following list of verbs may be helpful in guiding students to perform inquiring activities: explore, discover, create, prove, validate, construct, predict, experiment, investigate, etc. Inquiring activities usually requires some teaching aids. Teachers should therefore make proper preparations well before the

lesson so that adequate sets of aids are available. The following questions should be considered before organizing the inquiring activities in class:

- Will students be grouped when performing the activities? If yes, how many groups should be organized?
- How can we ensure that the right amount of guidance (in the form of hints or questions) is provided? (It should be noted that either insufficient or too much guidance will do no good to students.)
- When computers are available, what software could be used? Is there sufficient software for the whole class? If no, what can be done?

### Example:

### c. Conceptualizing Skills

**Design of the Innovation:** Conceptualizing involves organizing and reorganizing of knowledge through perceiving and thinking about particular experiences in order to

abstract patterns and ideas and generalize from the particular experiences.

**Description of the Innovation:** The formation of concepts involves classifying and abstracting of previous experiences. The particular problem of mathematics lies in its abstractness and generality. Abstract concepts cannot be communicated to students by a definition but only by arranging for him/her to encounter a suitable collection of examples. It follows that abstract concepts should be backed up by an abundance of mathematical and daily-life examples. Teachers need to provide students with a clear guidance to construct mathematical concepts from the examples and use these concepts to solve problems in unfamiliar situations. **When a new concept is introduced (like “All symmetrical triangles are similar.”), examples or counter-examples may be provided for illustration. Students may also be asked to explore the information relevant to the concept (like similar triangles or symmetrical triangles) and classify the similarities and differences in the examples.**

**Outcome of the Innovation:** It help students build up mathematical concepts, suitable examples or activities, which allow students to construct new concepts independently are necessary..

### Example:

Consider the straight line  $y = m \cdot x + c$ , When you teach this concept to the students in regular fashion, they tend to learn in the way you teach. They just know that it is a

straight line with few parameters called slope, x- intercept and y-intercept.

But, Teach this concept relating it to the real world scenarios. Teach them that  $y = m \cdot x + c$  is not just an equation to compute the value of 'y', instead teach them how this equation has formed a base for many future predictions that are happening in the science and technology like weather prediction, face recognition, traffic regulation, life expectancy etc.. Teach them the importance of calculating the value of 'y' and how it is helping in predicting the value of certain future 'y' based on past 'x' and 'y' values.

#### **d. Reasoning Skills**

**Design of the Innovation:** Reasoning is drawing conclusions from evidence, grounds or assumptions. It involves developing logical arguments to deduce or infer conclusions.

**Description of the Innovation:** Reasoning may be classified into inductive reasoning and deductive reasoning. Inductive reasoning works from specific observations to broader generalizations and theories while deductive reasoning moves from the other way round, that is, from the more general to the more specific. By its very nature, the inductive reasoning method is more open-ended and exploratory and the deductive one is narrower in nature and is usually concerned with testing or verifying hypotheses and theories. Therefore, finding the general term of a sequence like 1, 3, 5, 7, 9, ..... involves inductive reasoning while doing a geometric proof by applying a geometrical theorem (say, the corresponding angles of two similar triangles are equal)

involves deductive reasoning. Since reasoning is a fundamental aspect of mathematics, being able to reason is essential to the understanding of mathematical concepts. By making investigations and conjectures, developing and evaluating mathematical arguments, justifying results, etc., students are able to understand and appreciate the power of reasoning and produce proofs, which entail logical deductions of conclusions from theories and hypotheses. Reasoning, like other HOTS, cannot be taught in a single lesson. Instead, it is a habit of mind and should be a consistent part of students' mathematical experience. It is fostered or developed through a prolonged learning of mathematics in different contexts.

**Outcome of the Innovation:** To develop reasoning skills, students should be familiar with the following: 1. Sorting and classifying information, interpreting information and presenting results with pictures, diagrams, graphs, models, symbols and tables. 2. Describing, generalising, justifying patterns presented in a variety of forms and contexts, making conjectures, thinking flexibly, proving and refuting, recognising logical and illogical arguments, following a chain of reasoning, making deductions and demonstrating methods of mathematical proof (including proof by contradiction, counter-example and induction).

#### **Example:**

The following technique helps students to improve their higher order thinking skills in mathematics-

Challenging prompts such as “How do you know?” “Does it work for all cases?” and “Convince me” encourage student to verify or refute that is, prove or disprove conjectures.

“Is it true? Justify” tasks can be easily designed by classroom teachers. They are especially useful for confronting common misconceptions in many areas of the curriculum.

For example:  
 $26 + 47 = 613$  Is it true? Justify  
 $503 - 47 = 456$  Is it true? Justify  
 $1248 / 6 = 28$  Is it true? Justify  
 $(5 \times 7) + (2 \times 7) = 7 \times 7$  Is it true? Justify

For these examples students should be encouraged to use multiple methods to prove and disprove these claims. They can use estimation, diagrams, materials as well as mental strategies to prove or disprove to provide a logical argument.

## Implications

In these days of technical knowledge and scientific advancements, our education system is looking at developing the student's perspective towards science and technology. Maths will always be learned in a more interactive and innovative way rather than the traditional teaching methods. The innovative techniques when implemented in schools lead to the students progress in those areas and helps them to understand the concepts better.

*So, the intention of this paper is to suggest the Educational Authorities to include a chapter from class - 3 to class -10,*

*"Mathematics - Everyday Life" which describes the importance and applications*

*of all the remaining chapters that a Student learns in that academic year in terms of scientific uses and implementations.*

Few other techniques like twinning can also be implemented to expand and share the knowledge between the students of two different schools.

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STATE COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

TELANGANA STATE, HYDERABAD

DEPARTMENT OF MATHEMATICS AND SCIENCE

SEMINAR ON “ MATHEMATICAL TEACHING FOR EMERGING INDIA”

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2. **SUB THEME:** MATHEMATICS FOR DEVELOPMENT OF HIGHER ORDER THINKING SKILLS
3. **TITLE OF THE TOPIC:** MATHEMATICAL TEACHING FOR EMERGING INDIA
4. **OBJECTIVES:**

To introduce students to the process of mathematical investigation. By reading from different sources, working on problems, and conducting computer investigation, students will become more familiar with the process of mathematical inquiry, mathematical writing and presentation and develop Higher Order Thinking Skills.

5. **DESIGN OF THE INNOVATION:**

“We have got 21<sup>st</sup> century technology and speed colliding headon with 20<sup>th</sup> and 19<sup>th</sup> century institutions, rules and cultures” \_ Amory Lovins

“Young people today have lots of experience- interacting with the new technologies, but a lot less so of creating or expressing themselves with the new technologies. It is almost as if they can read but not write” – Mitch Resnick

Technology is not the answer for this but **Higher Order Thinking is.....**

Five Principles of Extraordinary Math teaching to inculcate **Higher Order Thinking Skills:**

- **Start with a question**
- **Students need time to struggle:** Thinking happens only when we have time to struggle. We deepen our curiosity in observation when time is given.
- **Teacher is not the answer key:** If teacher says the students “ I don’t know the answer , Let us find out” then Math becomes invention. Teach the students that concept.
- **Not Knowing is not Failure:** Say YES to the ideas of the Students. Saying YES is not the same thing as saying “ You have given right answer”.
- **PLAY:** Mathematics is not about following rules, it is about playing, it is mere Fun.

## 6. DESCRIPTION OF THE INNOVATION:

How can we engage students in higher order thinking skills?

Today's Technology is allowing students and Teachers to explore world in a homely way which prepares the student for JOBMARKET, help communication between the student and Teacher.

Key to engage the students in Higher Order Thinking is to first engage the Faculty . Engaging the students in Higher Order Thinking Skills is an experiential learning experience.

**Dan TapScott Says “To me, this is not an information age, it is an age of networked intelligence, and it is an age of vast promise”**

## 7. OUTCOME OF THE INNOVATION:

One of the main 21<sup>st</sup> century components that teachers want their students to use are higher-order thinking skills. This is when students use complex ways to think about what they are learning.

Higher-order thinking takes thinking to a whole new level. Students using it are understanding higher levels rather than just memorizing math facts. They would have to understand the facts, infer them, and connect them to other concepts.

Here are 10 teaching strategies to enhance higher-order thinking skills in students.

### 1. Teaching Strategies to Help Determine What Higher-Order Thinking is

Help students understand what higher-order thinking is. Explain to them what it is and why they need it. Help them understand their own strengths and challenges. You can do this by showing them how they can ask themselves good questions. That leads us to the next strategy.

For Example: Teacher makes a statement in the class

Qn. A triangle can have 2 right angles. (True / False).

Now, the students first questions himself what if a triangle has 2 right angles?

Then recalls Sum of the angles in a triangle is 180 degrees

Then adds  $90 + 90 = 180$  degrees

Then realizes that if the third angle is 0 degrees we cannot get a triangle.

So given statement is wrong.

### 2. Connect Concepts

Lead students through the process of how to connect one concept to another. By doing this you are teaching them to connect what they already know with what they are learning. This level of thinking will help students learn to make connections whenever it is possible, which will help them gain even more understanding.

Eg.

1. Introducing the concept of multiplication as repeated addition and division as repeated subtraction.

2.  $\log(\tan 45^\circ) = 0$  connects two different concepts viz., Logarithms and Trigonometry

### 3. Teach Students to Infer

Teach students to make inferences by giving them “Real-world” examples.. Then, ask them to make inferences.

For example:

Mr. Mochi owns a footwear business. He wants to expand his business, so he collected two random samples of 100 men regarding men's foot wear preference? Make an inference based on this data.

Student Sample	shoes	slippers	sandals	Total
#1	25	20	55	100
#2	30	10	60	100

Inference: Most Men prefer Sandals as footwear. Very rarely they use Slippers. So Mr.Mochi has to keep More no of Sandals in his shop to gain profits.

#### **4. Encourage Questioning**

A classroom where students feel free to ask questions without any negative reactions from their peers or their teachers is a classroom where students feel free to be creative. Encourage students to ask questions, and if for some reason you can't get to their question during class time, then show them how they can answer it themselves, or have them save the question until the following day.

#### **5. Use Graphic Organizers**

Graphic organizers provide students with a nice way to frame their thoughts in an organized manner. By drawing diagrams or mind maps, students are able to better connect concepts and see their relationships. This will help students develop a habit of connecting concepts.

For example:1. Mind map of Types of quadrilaterals

2. About a Triangle, an idea mapping

A triangle is a polygon with 3 straight sides and 3 angles. ( definition)

↓

You can make a triangle by folding a square piece of paper in half by opposite corners.

↓

A triangle is when you connect 3 straight lines to make a closed shape( Own words)

↓

Drawing picture of a triangle simply

#### **6. Teach Problem-Solving Strategies**

Teach students to use a step-by-step method for solving problems. This way of higher order thinking will help them solve problems faster and easier. Encourage students to use alternative methods to solve problems as well as offer them different problem-solving methods Like Solving Linear Equations algebraically and using Matrices.

#### **7. Encourage Creative Thinking**

Creative thinking is when students invent, imagine, and design what they are thinking. Using your creative senses help students process and understand information better. Research shows that when students utilize creative higher order thinking skills, it indeed increases their understanding. Encourage students to think "Outside of the box."

#### **8. Use Mind Movies**

When concepts that are being learned are hard, encourage students to create a movie in their mind. Teach them to close their eyes and picture it like a movie playing. This way of higher order thinking will truly help them understand in a powerful, unique way.

For example:

Write the similarities and differences between

- A rectangle and a parallelogram
- A parallelogram and a trapezium
- A kite (rhombus) and a square

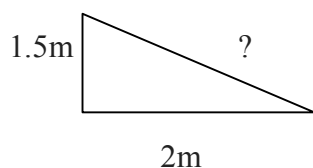


### 9. Teach Students to Elaborate Their Answers

Higher-order thinking requires students to really understand a concept, not repeat it or memorize it. Encourage students to elaborate their answers and talk about what they are learning. Ask parents to reinforce this at home, as well by asking the right questions that make students explain their answers in more detail, or to answer their child's question with a more detailed response.

Eg. Satvi wants to make a triangle-shaped box to fit in the corner of a square room. The sides of the box that touch the wall are given as 1.5 m and 2m long. How Long is the third side?

- Sol:
1. the box is a right angle triangle
  2. the two sides are 1.5m and 2m
  3. we have to find out the length of the third side i.e., hypotenuse.



So, Using Pythagoras theorem

$$\text{Hypotenuse}^2 = 1.5^2 + 2^2 = 2.25 + 4 = 6.25 \text{ m}$$

$$\text{Hypotenuse} = 2.5\text{m}$$

### 10. Teach QARs

Question-Answer-Relationships, or QARs, teach students to label the type of question that is being asked, then use that information to help them formulate an answer. Students must decipher if the answer can be found in a text or on the internet, or if they must rely on their own prior knowledge to answer it. This strategy has been found to be effective for higher-order thinking because students become more aware of the relationship between the information in a text and their prior knowledge, which helps them decipher which strategy to use when they need to seek an answer.

## 8. IMPLICATIONS:

Based on the result and discussion can be concluded that HOTS in mathematics learning is important. It aims to develop students' ability to analyze, evaluate, and create, so that students have the critical power and creativity that can be used to solve problems in everyday life. Some efforts should be made to improve students' HOTS in mathematics, namely: (1) engaging students in the activities of non-routine problem solving; (2) facilitate the students to develop the ability to analyze and evaluate (critical thinking) and the ability to create (creative thinking); and (3) encourage students to construct their own knowledge, so that learning becomes meaningful for students.

**Courage – curiosity – creativity are the beauty of Mathematical Thinking.**

## 9. References:

1. Janelle Cox. "Teaching Strategies that enhance Higher Order Thinking"
2. Mathematics Education -Trends and Challenges – School of Mathematics and Statistics, University of Hyderabad.
3. Mark Winegar: Higher Order thinking – Evaluation.
4. Dan Finkel, TEDxRainier: Five Principles of Extrordinary Math Teaching.

## MATHEMATICS TEACHING FOR EMERGING INDIA

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SUB THEME: MATHEMATICS FOR DEVELOPMENT OF HIGHER ORDER THINKING SKILLS

OBJECTIVE: TO DEVELOP HIGHER ORDER THINKING SKILLS IN STUDENTS FOR EXCELLING AS INDIVIDUALS IN THE 21<sup>st</sup> CENTUARY

DESIGN OF THE INNOVATION - TO DEVELOP A EDUCATION REFORM THAT INVOLVES THE LEARNING OF COMPLEX JUDGEMENTAL SKILLS SUCH AS CRITICAL THINKING AND PROBLEM SOLVING.

DESCRIPTION OF THE INNOVATION:-

Mathematics is essential in many fields. Mathematics is a growing field of knowledge, that is the number of applications also keep growing.

Mathematics is a cradle of creations. Certain qualities which are nurtured by maths are power of reasoning, creativity, abstract or spatial thinking ,critical thinking, problem solving ability and effective communicative skills.

Higher order thinking is a concept of education reform, which involves the learning of complex judgmental skills such as critical thinking and problem solving which can be developed through mathematics.

Higher order thinking skills is more difficult to learn or teach but also more valuable because such skills are likely to be usable.

As students move from elementary to middle to high school, they are asked by their teachers to do more and more with the information they have stored in their brains. These type of requests require accessing higher order thinking.

Most of us don't think about high order thinking, we just do it. But educators, parents, legislators have been thinking more about it.

## MATHEMATICS TEACHING FOR EMERGING INDIA

Higher order thinking takes thinking to higher levels than just restating the facts. It requires us to do something with the facts. We must understand them, infer from them, connect them to other facts and concepts, categorize them, manipulate them, put them together in new or novel ways, and apply them as we seek new solutions to new problems.

In India students level of learning is measured through their reading, writing and arithmetic abilities. Although prerequisite, but these are not sufficient for excelling as individuals in the 21<sup>st</sup> century, for preparing future leaders for the growth of nation.

### SKILL GAP IN INDIAN STUDENTS:

The Indian education eco system evaluates subject understanding and competence of students based on their ability to read and write and compute numbers. But in the 21<sup>st</sup> century, such evaluation is not sufficient. Considering the example of engineering, which is one of the most sought degree in India promises a potentially lucrative career. However, only 19% of engineering graduates in India are employable in jobs for which they received training. Students are not able to demonstrate correlation between having a qualification and performing well at work with in the current academic setting.

Therefore students enrolled in the educational institute must be enabled to acquire work place and life skills across multiple areas of knowledge for success. These values allow students to build an extraordinary profile which will help them receive many opportunities in life.

The culture of 21<sup>st</sup> century demands multiple areas of competency with in an individual.

The basic new skills that are helpful for being successful in higher education, career and life are:

### CURIOSITY AND IMAGINATION:

Curiosity leads to greater imagination and introduces creativity in the students thus creating pathway to delivering innovation.

### THINKING AND PROBLEM SOLVING:

Ability to ask good and thoughtful questions by critical thinking using the thoughts to solve the problems is much needed now.

### AGILITY AND ADOPTABILITY:

Recent years has seen a great shift to where people not only need to think independently but also devise agile strategies for the success of the team. They need to adopt different cultures, time zones and cooperate with other team members in a very potential manner to meet the strict deadlines.

### ACCESSING AND ANALYZING INFORMATION:

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Getting access to the information on the other hand is also important but should not just assume that all information is true. One must be able to validate credibility of the information before consuming and accepting it.

### EFFECTIVE ORAL AND WRITTEN COMMUNICATION:

Communication skills are a major factor highlighted in studies all over the world. These skills are only going to become more important as teams are increasingly composed of individuals from diverse cultures.

### COLLABORATION ACROSS NETWORKSWORLDWIDE AND LEADING BY INFLUENCE:

Progress in digital technologies implies that all project members need not be physically present at a place to execute the project. This is an era of virtual teams working on major infrastructural projects all over the world. These people are not present in the physical room ,instead from their own locations .They use variety of conference calling and web based technologies to conduct meeting and execute the project to success.

### AS MATHEMATICS TEACHER WE CAN DEVELOP HIGHER ORDER THINKING SKILLS IN OUR STUDENTS THROUGH THESE TEACHING STRATAGIES

Help students to determine what higher order thinking is and why they need it. Help them understand their own strengths and challenges.

### CONNECT CONCEPT

Lead students through the process of how to connect one concept to another.By doing this we are teaching them to connect what they all ready know with what they learn.

#### Examples:

Consider incorporating a small building in the classroom like sample house out of card board boxes or a small wooden boat from a kit to reteach maths related skills such as measuring estimating angles and following instructions.

Encourage students to play math challenges at the grocery store with their family. For a greater challenge encourage students to incorporate coupons, sales and adjusted pricing for bulk items.

### TEACH STUDENTS TO INFER

Teach students to make inferences by giving them real world examples.

#### Examples:

## MATHEMATICS TEACHING FOR EMERGING INDIA

- A tree diagram is effective for students to infer information and underlying meaning of the problem statement.
- A **number line** is another type of diagram that is being used increasingly by mathematicians

### ENCOURAGE QUESTIONING

Students feel free to ask questions without any negative reactions from their peers or their teachers is a class room where students feel to be creative .Encourage students to ask questions.

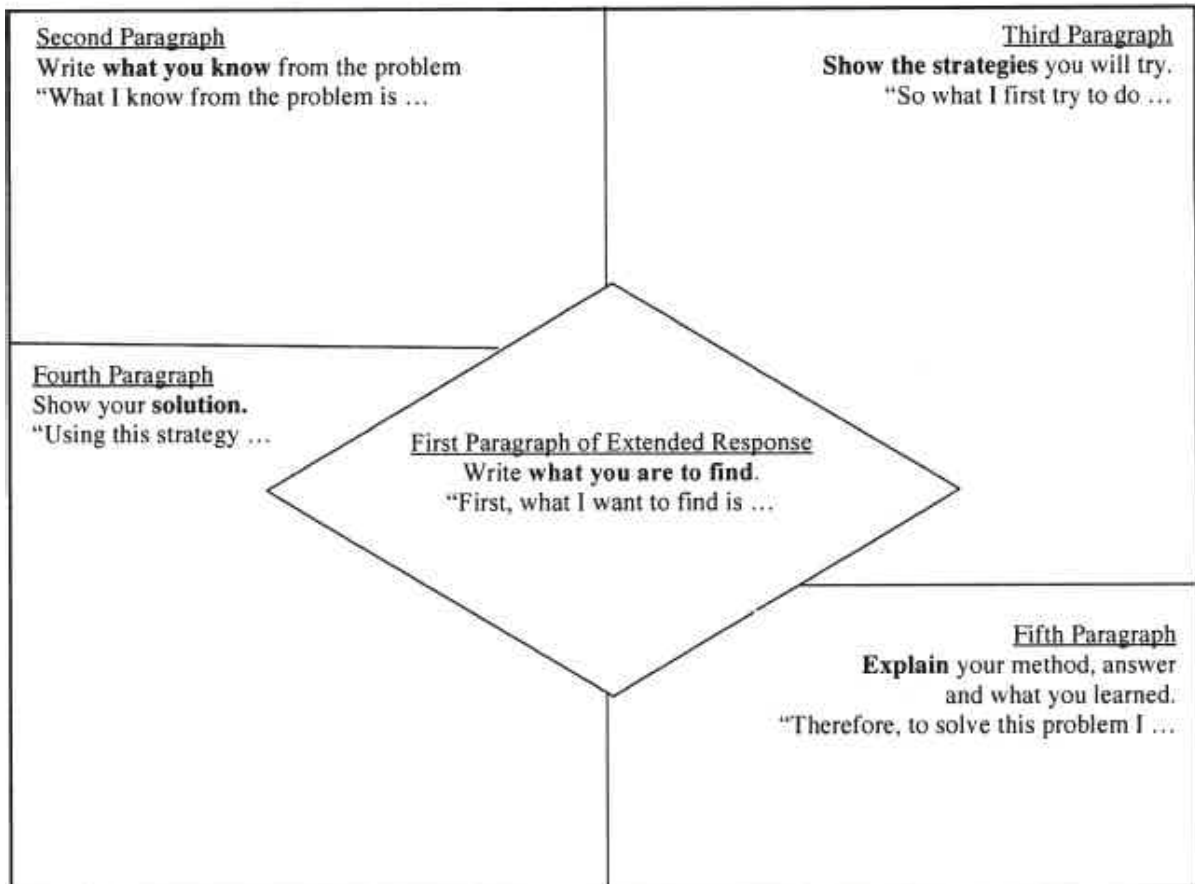
#### Examples:

- What does this problem make you think of?
- What skills have we learned in class that you can apply to this problem?
- What would be the first step in solving this problem? Or, tell your shoulder partner where you started.
- How did you solve this problem? Or, let's look at your work.
- Is there another way to solve this problem?
- Ask questions that include everyone
- No hands up rule
- Avoid teacher student ping pong
- Give students time to think
- Respond at greater length and greater confidence
- Avoid judging students responses
- Follow up students responses in ways that encourage deeper thinking

### USE GRAPHIC ORGANIZERS

Graphic organizers provide students with a nice way to frame their thoughts in an organized manner. By drawing or mind maps,students are able toconnect concepts better and see their relation ships .This will help students develop a habit of connecting concepts .

## MATHEMATICS TEACHING FOR EMERGING INDIA



### TEACH PROBLEM SOLVING STRATEGIES.

Teach students to use a step by step method for solving problems. Encourage students to use alternative methods to solve problems as well as offer them different problem solving methods.

#### Examples:

- Create a diagram
- Guess and check
- Use a table or make a list
- Logical reasoning
- Find a pattern
- Working backwards
- Solve an easier version first

### ENCOURAGE CREATIVE THINKING .

Creative thinking is where students invent, imagine and design what they are thinking .Research shows that when students utilize creative higher order thinking skills it indeed increases understanding. Encourage the students to think outside the box.

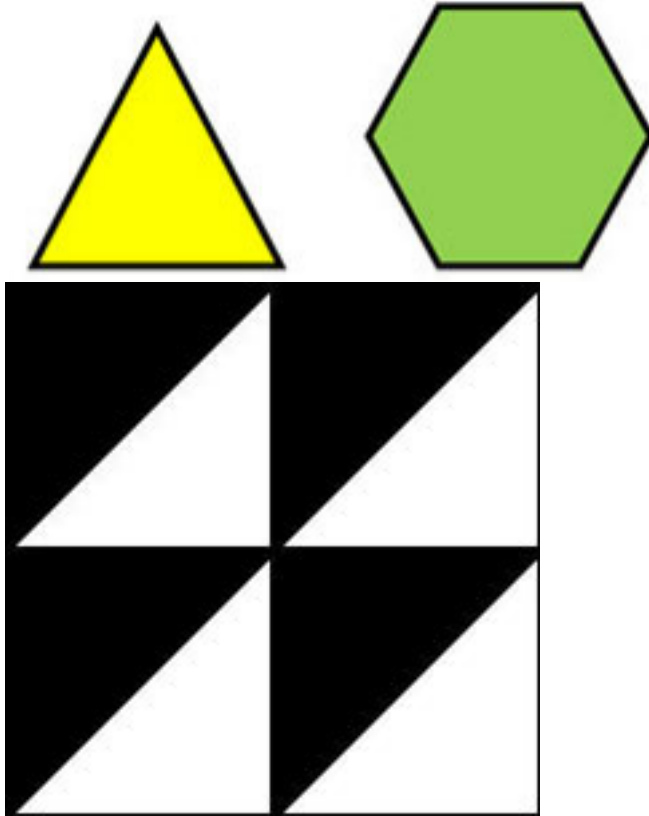
#### Examples:

## MATHEMATICS TEACHING FOR EMERGING INDIA

### **Activity 1: Students investigate triangles in polygons**

Ask your students the following:

- In how many ways can you divide each of the shapes in Figure 1 into right-angled triangles?



### **A square made up of eight right-angled triangles.**

- Do you think that all closed polygons can be formed by right angled triangles? Give reasons to justify your answer.
- Why do you think this activity asks you to investigate whether any closed polygons can be made up of right-angled triangles?

### USE MIND MOVIES.

When concepts are being learned, encourage students to create a movie in their mind.

#### Examples:

- Use 3 dimensional physical objects like a cube, cuboid, cone etc such that the students imagination is escalated.
- Describe conic sections in such a way that the student can visualize it in their mind.

### OUTCOME OF THE INNOVATION.

## MATHEMATICS TEACHING FOR EMERGING INDIA

WORLD NEEDS INDIVIDUALS WHO TAKE INITIATIVES, DEMONSTRATE LEADERSHIP IN EVERY ASPECT OF LIFE AND POSSESS SPIRIT TO OVERCOME CHALLENGES BY SEEKING OUT NEW OPPORTUNITIES, IDEAS, AND STRATEGIES FOR IMPROVEMENT. IT REQUIRES INNOVATORS AND POLICY MAKERS OF TOMORROW TO CREATE AND IMPROVE THE SOCIETY TO GREATER LEVELS THROUGH EXAMPLARY PRACTICES.



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Qualification	:	<b>M. A (Eng); PGDTE (CIEFL)</b> National level Resource Person in Vedic Mathematics.
Sub theme	:	Mathematics for Development of Higher order Thinking Skills
Title of the topic	:	VEDIC APPROACH in teaching Mathematics
Objective	:	Attaining conceptive confidence in Mathematics
Design of the innovations	:	VILOKANA Clubbed with ANURUPYENA
Description of the innovation	:	See below.
Outcome of the innovation	:	“
Implications of the innovation	:	“
Reference	:	Sri Sri Sri Bharati Krishna Swamiji's Vedic maths

Present day mathematics is the study of volumetric operations, application of formulae, and theorems and of course usage of terminology referred in books or web pages. In fact, mathematics, according to Indian ideology, is the result of the natural ability of OBSERVATION from the angle of one's KNOWN, FAMILIAR and convenient facets of life. They are called VILOKANA and ANURUPYENA in Sanskrit. Accordingly mathematics, GANIT in Sanskrit, is the outcome of the deep observation of things around us.

That's why it is said GANANEVA ITI GANITAH i.e. whatever is conceived or acquired through one and only one activity DEEP OBSERVATION, is only ganit.

It is quite evident with the world wide adaption of Indian DECIMAL SYSTEM, the TEN DIGIT/PLACE NOTATION SYSTEM of Recognisable and Conceivable quantities, that our Indian mathematical approach is far higher, better and of greater value than other approaches. It is the result of the human method of noting down all similar quantities with the TEN FINGERS he possesses.

It is also precisely conceivable that every living thing i.e.. human being, creature, plant, insect etc is blessed with the ABILITY of observation to get familiarised with the Environment, surroundings and occurrences. This ability only lets the living thing to prolong its life successfully and comfortably. From this ability VILOKANA only new concepts like SIMILAR or LIKE things, DISSIMILAR or UNLIKE things, Helpful or Harmful things, Feelings of enjoyment or struggle etc. have started coming into existence.

Slowly this VILOKANA only helped the human beings the specie with extraordinary ability of Vilokana to discover many, many new things, making the life more vivid, complex and wide spread. So Ganit according to Indian ideology is not just volumetric, operational and symbolic realisation but acquiring knowledge of every aspect of everything around us. Thus, SCIENCE, SOCIAL, LANGUAGE, ARTS and what not..... Everything is conceived very vividly, precisely, elaborately and enjoyably through this VILOKANA.

Vilokana helps realising the similarity, dissimilarity, relativity etc. And then, it is used not only in recognising these but the activities, Operations, their properties etc enlarging the GANIT. That's why our ancestors could measure the distances between planets, stars, far off places with mere visible observations only.

Let us now look into some examples from both normal Mathematical and Vedic angles, to understand this.

Normally people take a number to be itself precisely but don't try to analyse them with other numbers relatively. That's why whenever they need to operate with huge, complex and semi clear quantities, they get strained, bored or confused. But if they realise the relationship 'ANURUPATHA', of those numbers with the very familiar, well known and comfortable numbers, the same operations become so simple, enjoyable and very clear.

Please go through the following examples to understand this.

Addition:  $30,146 + 9,987$

In order to get the sum in normal method, one has to go on finding the sums of bigger digits like 7&6, 8&4, 9&1 all the while not forgetting the carrying aspect at every place. But, if one could smell 9,987 as a number just 13 less than 10,000, the addition becomes so simple as  $30,146 + 10,000$  and the sum turns out to be 40,146 which is 13 more than the expected answer. Thus, lessening 13 from that, one can easily tell it as 40,133, the correct expected total.

Isn't it?

Think!

Subtraction:  $27,46,002 - 39,888$

Again one has to use the borrowing process, that too many times if tries in normal way. But, a mere look at 39888 hints this number just 112 less than 40,000. See what happens:  $27,46,002 - 40,000 = 27,06,002$ . But it should come to mind that we have subtracted 112 more than given number. So, adding this 112 to the early conceived answer gives us 27,46,114 as the final answer

Check!

Enjoy!

Multiplication:

In normal method, one has to be very good at as many multiplication tables as possible to feel this operation easy. But, once the Multiplying numbers happen to be much bigger, one realises difficulty. Isn't it? See what happens if your Vilokana helps you to realise the Relativity of any one number with well known simple numbers.

Look at the examples given below.

Ex : 1. 1,3 8 7 x 9 9 8

Normal method

$$\begin{array}{r}
 1387 \\
 998 \\
 \hline
 11096 \\
 12483 * \\
 12483 ** \\
 \hline
 13,84,226 \\
 \hline
 \end{array}$$

Vilokana Method

$$\begin{array}{r}
 1387 \times (1000 - 2) \\
 1387000 \\
 \quad - 2774 \\
 \hline
 13,84,226 \\
 \hline
 \end{array}$$

Ex: 2 9,9 8 9 x 9,9 9 2

Normal method

$$\begin{array}{r}
 9989 \\
 9992 \\
 \hline
 19978 \\
 89901 * \\
 89901 ** \\
 89901 *** \\
 \hline
 9,98,10,088 \\
 \hline
 \end{array}$$

Vilokana Method

$$\begin{array}{r}
 9989 \dots > 10000 - 11 \\
 \times 9992 \dots > 10000 - 8 \\
 \hline
 10000 - (11 - 8) \quad \cdot \quad 0088 \\
 9981 \quad \cdot \quad 0088 \\
 \longrightarrow \quad 99810088
 \end{array}$$

**Vedic sutra**

*YAVADUNAM TAVADUNI KRITYA VARGANCHA YOJAYET*

Ex: 3 4,2 6 8 x 2 6

Vedic Vilokana Method

1/4<sup>th</sup> of (4,268) hundreds + (4,268) Because 26 is nothing but ¼th hundred +1

$$\begin{array}{r}
 1,06,700 \quad + \quad 4,268 \\
 = 1,10,968
 \end{array}$$

Observe !

Enjoy !

**PAPER PRESENTATION ON THE OCCASION OF NATIONAL MATHEMATICS DAY  
TO BE HELD ON 21/12/2019 AT SCERT, HYDERABAD, TELANGANA**

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**1. Sub Theme:**

Mathematics for Development of Higher Order Thinking Skills

**2. Title of the Topic:**

The Effectiveness of an Instructional Package on the content-specific Higher Order Thinking skills of students in Mathematics.

**3. Objectives:**

1. To study the effectiveness of Instructional kit/package in developing higher Order Thinking Skills on class X students.

**4. Design of Innovation:**

1. The Instructional kit contain model lesson plans and many example problems related to Higher Order Thinking Skills, and it included different strategies to explain the solution of the problems .
2. The study adopted the pre test and post test to equalent Group design.
3. The population of the study consisted of all X class students.

## **5. Description of the Innovation:**

The aim of Mathematics as a subject is the development of mathematical thinking that pursues clarity of thought, uses logical reasoning to justify logical conclusions, visualises and concretizes abstractions (NCF-2005). This aim can be fulfilled by integrating the Teaching strategies of all the above paradigms. The behaviorist strategies delivered in isolation can cause only "Knowledge" level learning, whereas together with cognitive strategies it can cater to the "comprehension" and the "application" level learning as well. Both the strategies if topped up with constructivist strategies can cause higher levels of learning that utilizes the mental skills of "Analysis, Synthesis, Evaluation and Creation". NCF-2005, position paper Teaching of mathematics, proposed pedagogical processes like formal problem solving, use of heuristics, estimation of approximation, generalization, visualisation, representation, reasoning and proof, making connections, mathematical communication to satisfy the goal of developing Mathematical Higher Order Thinking Skills (HOTS) among students. But since HOTS can not be developed without a strong foundation of conceptual and procedural knowledge which needs Lower Order Thinking Skills (LOTS); Every Mathematics topic need a well-integrated lesson plan with Teaching Strategies that cater to all the skills- Knowledge, Comprehension, Application, Analysis, Synthesis and Evaluation.

To develop the HOTS among the students a Instructional Package is prepared and it was used to study the impact of the Instructional Package, For this purpose all students of class X were made into 2 groups like Control group and Experimental group. Next randomly one of the groups was selected as control group and one group as Experimental group. The pre test was administered to both the groups. Their performance was note down in register. After the pre test, some of the chapters in X class math's like Real Numbers, Sets, Quadratic Equations. Polynomials and Statistics the control group was taught by the usual conventional method and Experimental group was by the Instructional package. After some days of instruction or Teaching learning process a Post Test was administered to the students of both the groups. The responses of the Post test were recorded and both the results were analyzed.

## **6. Out comes of the Innovation:**

The result indicate that the students exposed to the instructional package performed better in all areas like Comprehension, Application, Analysis and Synthesis in the post achievement test that focused on the questions relating to Higher Order Thinking abilities than that of the students exposed to the convention method.

## **7. Implications:**

The effectiveness of the developed package in terms of Higher Order thinking Skills especially for the concepts of Real Numbers, Sets, Quadratic Equations, Polynomials and Statistics indicate further implication of the teacher strategies used in the package. All the Secondary section can be designed using similar teaching strategies so that Teaching Learning can produce learning outcomes as envisaged by the NCF-2005 and RTE-2009 and as per the needs of the evolving generation.

## **8. Conclusion:**

Higher Order Thinking Skills and understandings are more difficult to learn and to teach, as they require more cognitive processing and different forms of instruction. In this generation “Students are not as receivers of information but as users of information.” By normal method of Teaching Learning Process it is difficult to improve Higher Order Thinking Skills among the students in mathematics. So the Teachers are advised to use different methods, techniques and strategies to develop these skills including problem solving, analytical thinking, critical thinking, creativity and to develop curiosity, interest, towards mathematics. There are many ways to enhance Higher Order Thinking Skills for students, however similar to other skills; Higher Order Skills require training, practice and patience. Learning environment that students have been required not only to remember information, but also they may resist with assessment questions that are not taken verbatim from the book, students may be required solving real-life problems to learn mathematics. However, whether this instruction for developing Higher Order Thinking succeeds or not, it depends very much on Teacher. Our study results once confirm the necessity to pay attention and focus on training and retraining of instruction for Higher-Order thinking skills for secondary school students’

## **9. References:**

1. Blooms Taxonomy
2. Collins R (April, 2014) skills for the 21st century, Teaching Higher-Order Thinking Curriculum and leadership journal.
3. Maths Text Book of Class X of Telangana State and NCERT.

## **ITEMS TO BE COVERED IN THE INSTRUCTIONAL KIT/ PACKAGE**

### **INSTRUCTIONS TO THE TEACHERS:**

1. Teaching Strategies that enhance Higher Order Thinking skills

- a) Connect Concepts
- b) Teach Students to infer
- c) Encourage Questioning
- d) Use Graphic Organize
- e) Teach Problem- Solving strategies
- f) Encourage Creative Thinking
- g) Use mind Movie
- h) Teach Students to Elaborate Their Answers
- i) Teach QARs (Question -Answer-Relationship)

When children are solving given problems, in the meanwhile by asking these type of questions we may surely develop Higher Order Thinking Skills among the children.

Those questions are:

- a) What tools will you need?
- b) How did you tackle similar problems?
- c) How did you get the answer?
- d) Do you agree or not?
- e) Can you think of a counter example?

### **MODEL QUESTIONS TO BE ASKED FOR DEVELOPMENT OF HOTS:**

1. A Peacock is sitting on the top of a pillar, which is 9m high from a point, 27m away from the bottom of the pillar a snake is coming to its hole at the base of the pillar, seeing the snake the peacock pounces on it. If their speeds are equal, at what distance from the hole is the snake caught?
2. How can we say that every null set is subset of every set?
3. Can we find G.C.D of 0.25 and 1.5 by using Euclid Division method?

**Thank you so much for giving me this opportunity.**

Write up for the SEMINOR to the SCERT on Dec 21st ,2019

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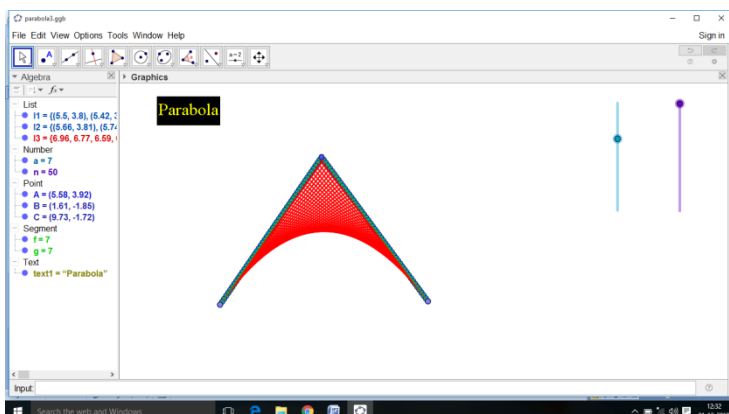
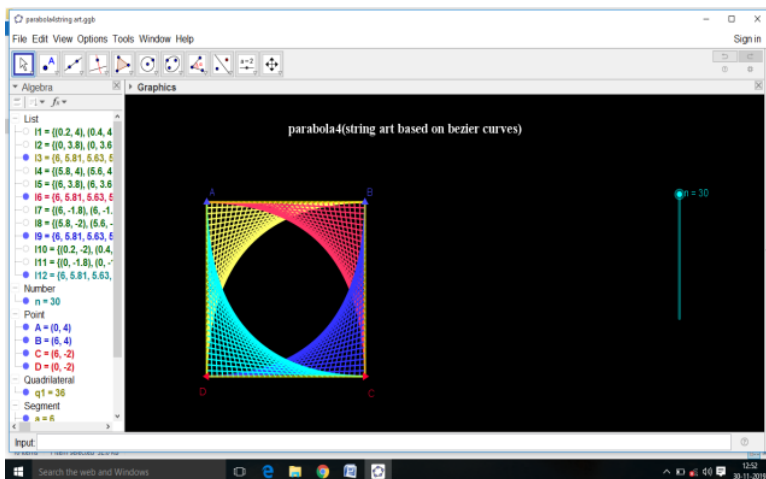


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**Sub Theme:** Mathematics developing for HIGHER ORDER THINKING

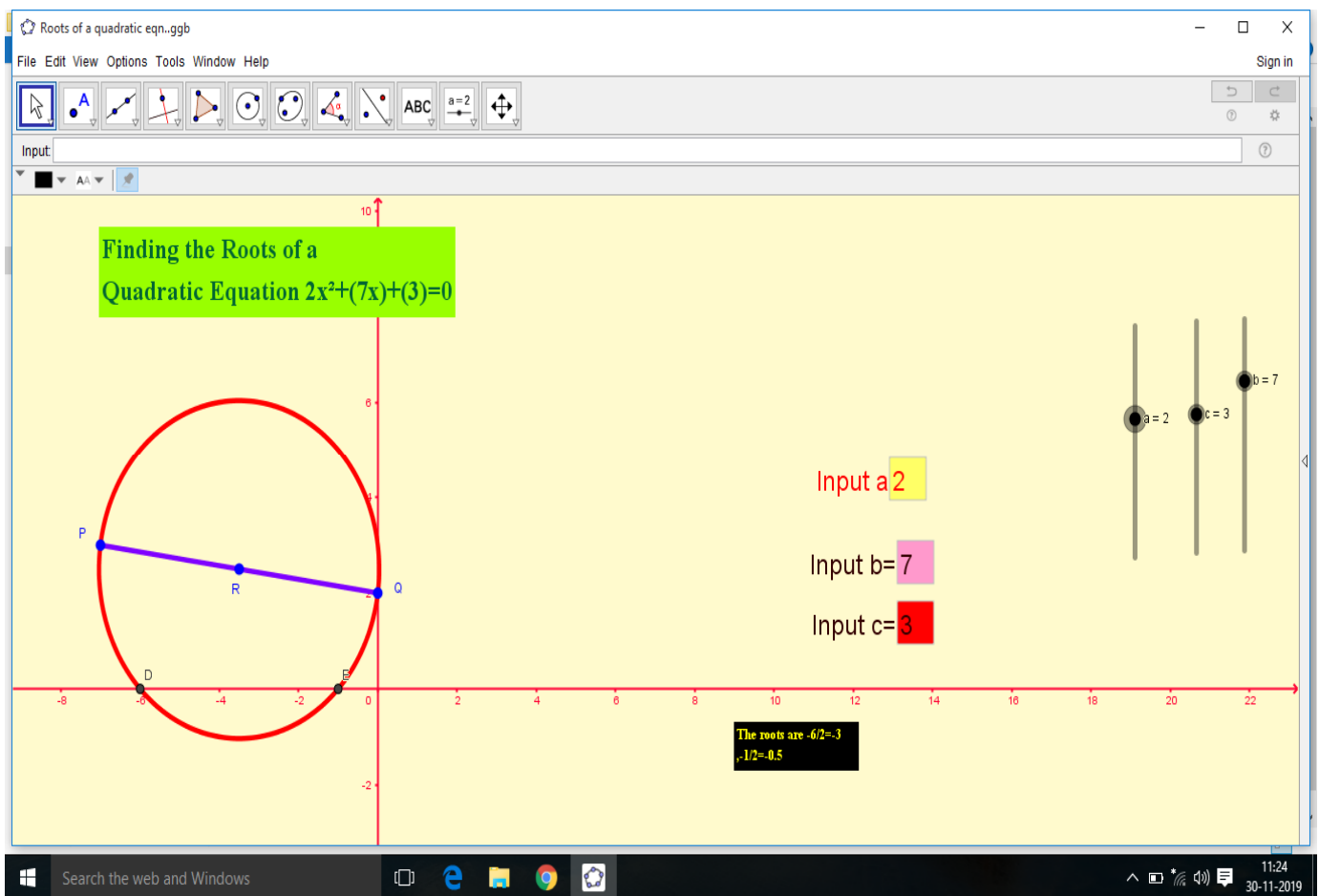
**Title of the Topic:** Finding the Roots of a Quadratic Equation by drawing a Circle

**Objectives:** Finding the Roots of a Quadratic Equation is possible in 2 methods

1. Algebraic Method
2. Graphical Method

In Graphical method we can find the roots of a Quadratic Equation by drawing a Parabola. The x- coordinates of the intersection points of the Parabola and to the X axis are called Roots of the Equation. But, here without drawing a Parabola we can find the Roots by drawing a Circle.

**Design of the Innovation :**



### **Description of the Innovation :**

- i) Draw a Graph by taking suitable scale.
- ii) Paste the graph on the Card Board
- iii) Mark a Point Q(0,1) and fix a paper Pin at Q and Tie one end of the thread of length nearly 30 cm.

**\*Let us find the Roots of a Quadratic Equation of the form  $x^2 - ax + b = 0$ \***

### **NOTE:**

**Here the coefficient of  $x^2$  should be 1 and the coefficient of the x term is negative.**

On the Graph Board Locate the Point Q(0,1) with a pin and denote P(a,b) on the board and join. Find the midpoint of PQ and denote it as R(a/2, 1+b/2). By taking PR as Radius draw a Circle. Denote the intersection Points of the Circle to the X axis.

The X co ordinates of these intersection points will be the Roots of the given Quadratic Equation.

By using this graph we can say the Nature of the Roots.

\*If the Circle touches the X axis at two points then the Roots are Real and Distinct

\*If the Circle touches at only one point then the Roots are Real and equal

\*If the Circle doesn't touch the X axis then the Roots are not Real

The Principle Behind : The equation of a Circle whose end Points of the Diameter (a,b) and (c,d) is  **$(x-a)(x-c) + (y-b)(y-d) = 0$**

Here the End points of the Diameter are (0,1) ,(a,b)

Then the Equation of the Circle becomes

$$(x-0)(x-a) + (y-1)(y-b) = 0$$

$$x^2 - ax + y^2 - y(b+1) + b = 0$$

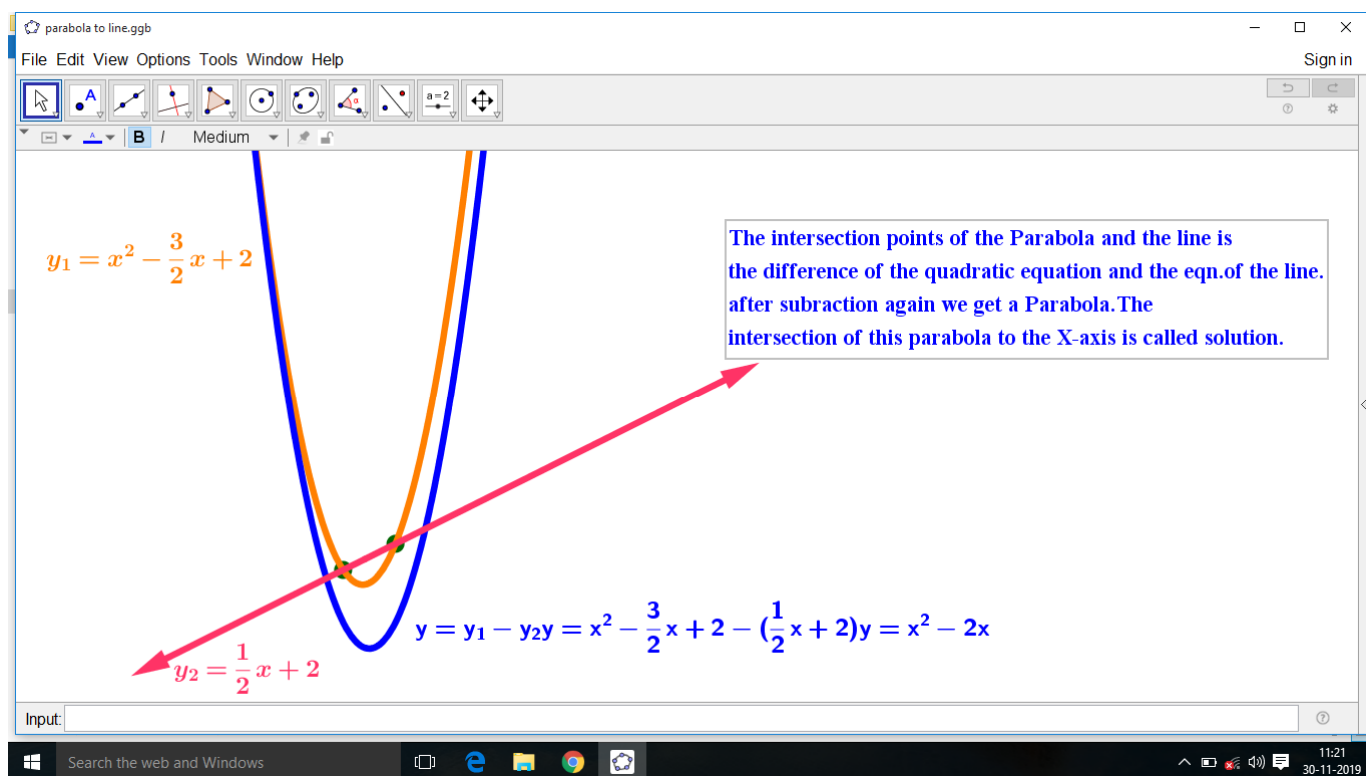
The intersecting the X-axis, so  $y=0$

Substitute  $y=0$  in the above equation, we get  **$x^2 - ax + b = 0$**

Hence, this is the form of a Quadratic equation , whom we have to find the Roots.

**Out come of the Innovation:** Now we will find the Roots of a Quadratic equation of the generalised form  $ax^2+bx+c=0$

- i) Denote P(0,a) and Q(-b,c)
- ii) Join PQ and find the Midpoint R.
- iii) With radius PR draw a Circle.
- iv) Denote the intersecting points of the Circle to the X axis
- v) Divide the X co ordinates with the co efficient of  $x^2$  i.e. 'a'
- vi) They are the Roots.



**Implications :** To find the solution to the Quadratic equation and the Linear equation is find the algebraic difference , we get the Quadratic Equation and draw the circle to the resultant Expression.

The x co ordinates of the circle will be the solution.

**References :** Intermediate Mathematics Text book and the Old SSC Mathematics Text book

**Note:** Graphical representation is drawn by using Geogebra

# MATHEMATICS SEMINAR

**Theme : “Mathematics Teaching for Emerging India”.**

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**Sub Theme : Mathematics for Development of Higher order thinking Skills”**

**Title of the Topic** : Formula / Generalization for finding the product of 142857 and any number without actual multiplication.

**Objectives :**

- (i) To inculcate / to improve higher order thinking skills in Mathematics among teachers and students.
- (ii) To improve innovative ideas with simple instances.

observe the ~~the~~ results of  $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \dots, \frac{6}{7}$  which are following a cyclic pattern.

$$\frac{1}{7} = 0.\overline{142857}$$

$$\frac{2}{7} = 0.\overline{285714}$$

$$\frac{3}{7} = 0.\overline{428571}$$

$$\frac{4}{7} = 0.\overline{571428}$$

$$\frac{5}{7} = 0.\overline{714285}$$

$$\frac{6}{7} = 0.\overline{857142}$$

Here the speciality I observed is that there is a cyclic pattern of digits of the product results of 142857 with 1, 2, 3, 4, 5, 6. The products are as follows.

$$\begin{aligned} 142857 \times 1 &= 142857 \rightarrow P_1; \text{ Let us assume that } 142857 = P_1 \\ 142857 \times 2 &= 285714 \rightarrow P_2 \\ 142857 \times 3 &= 428571 \rightarrow P_3 \\ 142857 \times 4 &= 571428 \rightarrow P_4 \\ 142857 \times 5 &= 714285 \rightarrow P_5 \\ 142857 \times 6 &= 857142 \rightarrow P_6 \\ 142857 \times 7 &= 999999 \rightarrow P_7 \end{aligned}$$

Product with 1;  $142857 \times 1 = 142857$

Product with 2;  $142857 \times 2 =$  In this product multiply  $7 \times 2$  we get 14.

If we re-write the number 142857 as replacing 2857 before the 14 then we get 285714 which is required result.

Product with 3:  $142857 \times 3$

Multiply  $7 \times 3 = 21$

Replace 42857 before the last digit 1.  
We get 428571, which is required result of  $142857 \times 3$ .

Product with 4:  $142857 \times 4$

Multiply 7 by 4 ;  $7 \times 4 = 28$

Replace 57 before 1428 we get 571428  
which is required result.

In the same way  $\uparrow 142857 \times 5 = 714285$

$\uparrow 142857 \times 6 = 857142$

Product with 7: In the product of 142857 by 7

we get interesting number i.e.,  
 $142857 \times 7 = 999999$ .

⇒ It means for every 7 digits (or) for every successive 7 numbers the product results are being changed.

Let us observe the change in cyclic patterns of the next 7 products of 142857.

Products of 142857 and 8, 9, 10, 11, 12, 13, 14 :- [ '1' replacing series ]

(i)  $142857 \times 8 = 1142856$

To get this product Subtract '1' from  $142857 \xrightarrow{P_1}$  and replace '1' before the result 142856

⇒  $142857 - 1 = 142856$

⇒  $\underline{1} 142856$

\*\* ⇒ In other words  $\underline{1(P_1-1)}$

(ii)  $142857 \times 9 = 1285713$

To get this product Subtract '1' from 285714 ( $P_2$ )

⇒  $285714 - 1 = 285713$

Replace 1 before it

⇒ 1285713.

\*\* In other words  $\underline{1(P_2-1)}$ .



(iii)  $142857 \times 10 = 1428570$ ; Similarly do  $P_3 - 1$   
 $= 428571 - 1$   
 $= 428570$   
 Replace 1 before 428570  
 $= 1428570$ .

(iv)  $142857 \times 11 = 1571427$ ; It can be obtained from  $1[P_4 - 1]$   
 $1[571428 - 1]$   
 $= 1571427$

(v)  $142857 \times 12 = 1(P_5 - 1) = 1(714285 - 1) = 1714284$

(vi)  $142857 \times 13 = 1(P_6 - 1) = 1(857142 - 1) = 1857141$ .

In the same way

(vii)  $142857 \times 14 = 1(P_7 - 1) = 1(999999 - 1) = 1999998$

We can conclude that the above 7 successive products are forming a cyclic pattern.

Products of 142857 with 15, 16, ... 21 :- [2 replacing series]

Observe the products of 142857 by 15, 16, 17, 18, 19, 20 and 21 which are also following a specific rule:

(i)  $142857 \times 15 = 2(P_1 - 2) = 2(142857 - 2) = 2142855$

(ii)  $142857 \times 16 = 2(P_2 - 2) = 2(285714 - 2) = 2285712$

(iii)  $142857 \times 17 = 2(P_3 - 2) = 2(428571 - 2) = 2428569$

(iv)  $142857 \times 18 = 2(P_4 - 2) = 2(571428 - 2) = 2571426$

(v)  $142857 \times 19 = 2(P_5 - 2) = 2(714285 - 2) = 2714283$

(vi)  $142857 \times 20 = 2(P_6 - 2) = 2(857142 - 2) = 2857140$

(vii)  $142857 \times 21 = 2(P_7 - 2) = 2(999999 - 2) = 2999997$ .

In the same way next 7 successive products also follow the cyclic repetition of digits.

From the above cyclic repetitions I concluded and derived a new generalisation for finding the products of 142857 by any natural number.

In the above products we obtained specific results for 7 multiples ~~and~~ such as 7, 14, 21, ... and non-multiples of 7.

So I generalised a new formula separately for multiples of 7 and non-multiples of 7.

### Outcome of Innovation:

(i)  $142857 \times$  (Non Multiple of 7)

Let  $x$  is non multiple of 7

$\Rightarrow x$  is ~~obtained~~ ~~by~~ to be divided by 7

so that  $x = 7k + m$ ; where  $m = 1, 2, 3, 4, 5, 6$

here Digits of  $k$  is to be replaced before the difference of  $P_m - k$

$$\therefore \underline{142857 \times x} = k [P_m - k]$$

(ii)  $142857 \times$  (Multiple of 7)

Let  $7M =$  multiple of 7.

Digits of  $(M-1)$  is to be replaced before the difference of  $P_7 - (M-1)$ .

$$\therefore \underline{142857 \times 7M} = (M-1) [P_7 - (M-1)]$$





# MATHEMATICS TEACHING FOR EMERGING INDIA

Mathematics for development of Higher order  
thinking

## Abstract

This resource illustrates practical activities to improve Learning and teaching skills and support improvements by utilizing higher order thinking skills

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Theme: - Mathematics Teaching for Emerging India

Sub Theme: - Mathematics for development of Higher order thinking

- problem solving: seeking and identifying strategies and reasoning
- comprehension and interpretation of statistics
- flexibility of thinking
- using and understanding appropriate mathematical vocabulary
- identifying the steps and using a number of operations
- realising the importance of accurate calculations
- applying inverse operations

## Activities to support learning and teaching

### Practical activity 1 - Hinge questions

The Mathematics Excellence Group advocates strongly the planning of questions into lesson preparation. Such questions have been called 'hinge questions'. The idea is that the teacher plans every lesson with a 'hinge'; a point in the lesson when the teacher can check on student understanding, and then decide what to do next. 'Hinge' questions are typically designed to test learners' understanding of one important concept in a lesson—one that is critical for pupils to comprehend before the teacher moves on in the lesson.

### *Hinge Questions?*

*Using questioning to determine how a lesson will progress is something that good teachers do instinctively. So, what is different about a hinge question?*

*A hinge question is planned within a lesson to gauge the level of understanding, the depth of thinking and hence to determine the next stage of the lesson.*

- The planning of the question prior to the lesson is essential
- The question should be asked about midway in the lesson to allow time to address the issues
- Everyone in the class **MUST** respond to the question within two minutes
- The teacher must be able to assess the results within less than a minute

### *The Benefits*

- It develops thinking skills
- Improves reasoning skills
- Encourages discussion, debate and explanation
- Reveals misconceptions

### *Possible ways to conduct the technique:*

*Question is put to the class – on board, orally, or using SMART response*

*The question may be put in the form of a multiple choice but need not be. The important point to ensure is that the likely responses are thought out by the teacher prior to the lesson allowing immediate feedback to take place*

*Pupils could respond by using 'show me boards' or by the handset if using SMART*

*The emphasis must be on the quick response by the pupils*

*Where opinion is divided, pupils construct their case to persuade others of their point of view*

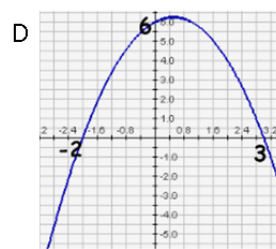
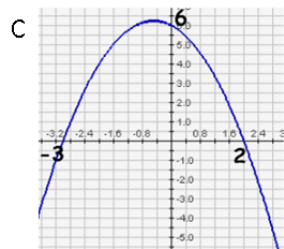
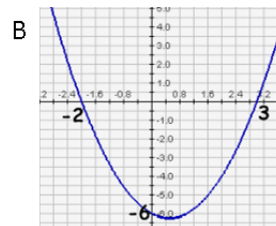
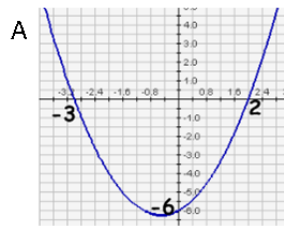
### Hinge Questions

Hinge Questions: example 1			
The gradient of the line joining the points (2,3) and (5,-7) is?  A $\frac{3}{-10}$ B $-\frac{10}{3}$ C $-\frac{4}{7}$ D $\frac{7}{-4}$			I have discussed ways to describe the slope of a line, <b>can interpret the definition of gradient and can use it to make relevant calculations</b> , interpreting my answer for the context of the problem.
Correct Answer	B: $m = \frac{y_2 - y_1}{x_2 - x_1}$		
Thinking for wrong answers	A: x's on top and y's on bottom	C: added instead of subtracting	D: <b>added</b> x's on top and y's on bottom
How do you deal with the incorrect answers?	Go back to diagram and look at definition of gradient		

Hinge Questions: example 2			
The distance between the two points (3, 5) and (8, 7) is?  A $\sqrt{29}$ B $\sqrt{21}$ C $\sqrt{265}$ D 7			I have explored the relationships that exist between the sides, or sides and angles, in right-angled triangles and can select and use an appropriate strategy to solve related problems, interpreting my answer for the context.
Correct Answer	A: $\text{distance}^2 = (8 - 3)^2 + (7 - 5)^2$ $= 5^2 + 2^2$ $= 25 + 4$ $= 29$ $\text{distance} = \sqrt{29}$		
Thinking for wrong answers	B: subtracting the squares	C: adding the coordinates then squaring	D: $\text{distance} = (8 - 3) + (7 - 5)$ $= 5 + 2$ $= 7$ Not squaring anything

### Hinge Questions: example 3

$f(x) = (x - 3)(x + 2)$ .  
Which diagram shows the graph of this function?



Correct Answer	B: Working for correct solution		
Thinking for wrong answers	A: Explanation	C: explanation	D: explanation
How do you deal with the incorrect answers?	Explanation		

### Hinge Questions: blank

Insert question			
Correct Answer	Working for correct solution		
Thinking for wrong answers	Explanation	explanation	explanation
How do you deal with the incorrect answers?	Explanation		

## Practical activity 2

# Think - Pair - Share

Think-Pair -Share is a powerful way to promote thinking and listening skills in learners and aid the staff in making each learner accountable for their learning and to get a quick and concise idea of who in the class understands the problem being discussed.

The problem can be anything; it is the timed sections of the activity and what is going on in the class that is the important part of this technique.

'1 for me, 2 for you'

First of all, the staff should present the problem and using a countdown clock give the learners a minute to read, think and try and solve the problem all on their own (this minute's silence can also be a good settler for a class at beginning of a lesson). When the buzzer signals that the minute is up, another minute is given for the learners to discuss the problem in pairs. It is a good idea here to tell the learners that the minute will be split equally, one partner alone speaks for 30 seconds, and the other must listen intently.

"I may ask you to explain what your partners views were and you must use their words not your own"


Once both have had a fair and equal chance to explain their thinking and present their solution to the other, the staff can decide, after listening into the pairs as they walk round the classroom, if a third minute is needed for the pairs to form groups of four to further discuss or if the problem has been tackled well enough in pairs.

The staff can now get a feel of the class' understanding of the problem using traffic lights or thumbs or take a quick vote on possible answers from the pairs. The staff can then explore further and ask individuals to explain what their pair thinks or indeed what their partner thinks ensuring everyone in the class is accountable and has been taking part in the task. Here the staff can explore the wrong answers, the alternative solutions and check learner understanding.

## Example - Block of butter

A block of butter is cut into two identical blocks, x and y.

Is the total surface area of x more than half, exactly half or less than half of the original block?

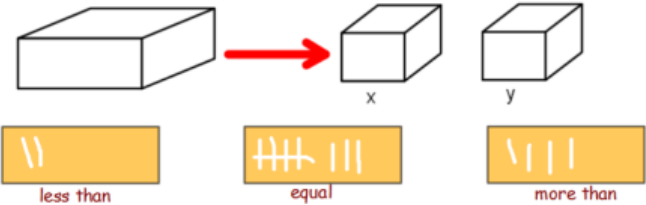


The diagram shows a 3D rectangular block on the left. A red arrow points from it to two smaller 3D rectangular blocks on the right, labeled 'x' and 'y'. The blocks 'x' and 'y' are identical to each other and represent half the volume of the original block.

The staff then adds boxes to take a class vote

A block of butter is cut into two identical blocks, x and y.

Is the total surface area of x more than half, exactly half or less than half of the original block?



The diagram is identical to the one above, but with three yellow boxes below the blocks 'x' and 'y'. The first box is labeled 'less than' and contains two vertical lines. The second box is labeled 'equal' and contains five vertical lines. The third box is labeled 'more than' and contains four vertical lines.

The staff can now continue to question the class

- "Why did your pair vote more than?"
- "Can you explain why Sophie said she thought it was equal?"
- "Is there another way we can think about the way to solve this problem?"

The lower order thinking for the learners is the remembering what surface area means and understanding how the surface area is calculated.

The chance learners have to think and to talk about the problem gives them the chance to move into the higher order thinking skills needed to solve the problem. Analysing what happens to the surface area as the block is cut in half, how the shape has changed and then evaluating the increase in surface area as this a new surface on each half block that weren't there before.

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1. National Improvement Hub, Scotland
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**STATE COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING  
TELANGANA STATE, HYDERABAD**

**DEPARTMENT OF MATHEMATICS AND SCIENCE**

**STATE LEVEL SEMINAR ON “MATHEMATICS TEACHING FOR EMERGING  
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**MAIN THEME**

- **Mathematics for development of higher order thinking skills**
- **Higher order thinking and 21<sup>st</sup> Century Skills**
  
- **Introduction of (H.O.T) and why?**
  
- **Why do we teach higher order thinking**
  
- **How do we teach (H.O.T) skills**
  
- **The high investment of higher order thinking**

**OBJECTIVE/OBJECTIVES**

**High order thinking skills**

**Creating new methods**

↓

**Evaluating new methods**

↓

**Analysing new methods**

↓

**Applying new methods**

↓

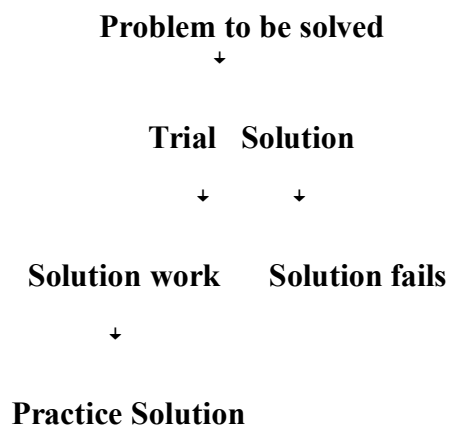
**Understand new methods**

↓

**Remembering new methods**

## SPECIFIC OBJECTIVES

- Specific objectives for students ( abbreviation) learner
- S- Feel really fun and enjoy the learning mathematics subject
- T- are able to understand abstract thinks easily by the usage of technology
- U- are interested to learn through mobile and computer
- D- Tabs and projector as the subject get register in there minds
- E- Enironmental with energetic power
- N- are enriched with different models of learning which enriches their knowledge
- T-Regularity and punctuality is increased as they are interested in multi sector learning
- Design of the innovation
- (Trial and Error method):



- A kind of learning in which one response after is tried (practice) and rejected as ineffective (errors) until eventually a successful response is made  
By –Edward thorndike

## ➤ DESCRIPTION OF INNOVATION

### **Newest Teaching Methods & New Tools**

Interactive teaching and learning method using the latest technology was world wide tried and accepted method as it demonds the attention of all demain of learning and long term memory based.

#### THEN

Pen  
Chalk board  
White Board  
Telephone  
Fox  
Library

#### NOW

Google  
Date projector  
Moves

#### NEXT

Vitual reality  
Rellsable paper  
Sematic web  
I pad Apple  
Computer



## **OUT COME OF INNOVATION**

- **Students are able understand the content easily where teacher is just a facilitator**
- **Students are able to understand the difficult concepts**
- **Students are able learn standard memory based knowledge and subject concept maths.**

## **IMPLICATION**

- **Students in turn learn the subject/concept using technology at formative age**
- **Students will not depend on other for usage of technology at all evels of there learning**
- **Students are equipped with technology along with subject knowledge**
- **They need not waste there time in learning in this technology skill and for searching this livelihood after education**

**Reference/overall outcome knowledge**

**Connect the concepts**

**Teach students to infar**

**Encourage the questioning**

**Encourage the creative thinking**

# A SEMINAR PAPER ON “DEVELOPMENT OF HIGHER ORDER THINKING THROUGH OPEN-ENDED QUESTIONS”

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**Theme: Mathematics Teaching For Emerging India**

**Sub-theme: Mathematics for Development of Higher Order Thinking Skills**

**Topic: Development of Higher Order Thinking through Open-ended Questions**

## 1. OBJECTIVES:

- To understand about Higher Order Thinking
- To develop the Concept of Open-ended Questions
- To prepare Open-ended Questions based on Close-ended type questions

## 2. INTRODUCTION:

Think about thinking is the most important. The educators, parents and policymakers have been thinking more about thinking. The thinking is need how the teachers want to teach students to think. As the students move from primary to secondary school, they are asked by their teachers to do more and more with the information they have stored in their brains. They may ask students to write a new ending for a book they've been reading or they may ask why a certain character in the story behaved in a particular way.

If the students are studying arithmetic, students might be asked to think and get reason for the number patterns. If they are studying geometry, students might be asked to think and how to get actual areas of given field which are different shapes. If they are studying algebra, students might be asked to think and equations by the given statements. These types of requests require higher order thinking. Higher order thinking may seem easy for some students, but difficult for others. The most of the higher order thinking skills can be learned and with practice a person's higher order thinking skill level can increase.

### 3. WHAT IS HIGHER ORDER THINKING?

Higher order thinking involves the learning of complex skills such as critical thinking and problem solving. These 'higher order', requires different learning and teaching methods than the learning of facts and concepts. Higher order skills are skills involving creation, analysis, evaluation and synthesis.

Higher order thinking is thinking on a level that is higher than the memorizing facts or telling something. When a person memorizes, he gives back the information without having to think about it, we call it as rote memory. That's like a robot which is need a program to do, but it doesn't think for itself.

Higher order thinking ("HOT") takes thinking to higher levels than restating the facts. HOT requires that we do something with the facts. We must understand them, infer from them, connect them to other facts and concepts, categorize them, manipulate them, put them together in new or novel ways and apply them as we seek new solutions to the new problems.

### 4. THE CONCEPT OF OPEN-ENDED QUESTIONS

One way of categorizing questions is to describe them as either 'open-ended' or 'close-ended'. Close-ended questions are those that simply require an answer or a response to be given from memory, such as a description of a situation or object or the reproduction of a skill.

- Ex: What is even prime number?

Open questions are those that require a student to think more deeply and to give a response which involves more than recalling a fact or reproducing a skill.

- Ex: Write other than ten prime even numbers.

Teachers are usually skilled at asking open questions in learning areas such as language or social studies. For example, teachers often ask students to interpret situations or justify opinions. However, in mathematics lessons closed questions are much more common.

Questions which encourage students to do more than recall known facts have the potential to stimulate higher levels of thinking. To emphasize problem solving, application and the development of a variety of thinking skills it is vital that we pay more attention to improving our questioning in mathematics lessons. Teachers should use questions which develop their students' higher levels of thinking. So, We call Open-Ended question as a 'good' question.

The goals of education are for our students to think, to learn, to analyse, to criticise and to be

able to solve unfamiliar problems. Then it follows that ‘good’ questions should be part of the instruction of all teachers of mathematics. In this paper how to create ‘good’ questions as open ended form and discussed some practical ideas for using them in the classroom.

## 5. THE PROCESS FOR PREPARATION OF OPEN-ENDED QUESTIONS

Now, we discuss two processes for preparation of open-ended questions.

### PROCESS 1: WORKING BACKWARDS

We can create open-ended questions through one process as working backwards.

This is a three-step process as follows:

Steps	Description	Example
Step 1	Identify a topic.	averages
Step 2	Think of a closed question and write down the answer.	The ages of the Rakesh family members are aged 5, 8, 12, 35 and 40. What is their average age? The answer is 20.
Step 3	Make up a question that includes (or speaks) the answer.	There are five members in a family. Their average age is 20. How old might the family members be?

Some more examples to create open-ended questions through working backwards:

	Step 1	Step 2	Step 3
Example	Identify a topic	<b>Think of an answer /</b> Think of a closed question and write down the answer.	<b>Make up a question that includes the answer</b>
1	Money Counting	How many Rs 10 coins can you get for Rs 50? The answer is 5 coins.	How many different ways can you get 5 coins to make Rs 50?
2	Counting	6 Tables	Ravi counted something as 6 exactly in his class room. What might he has counted?
3	Area	10 cm <sup>2</sup>	How many triangles can you draw each with an area of 6 cm <sup>2</sup> ?
4	Fractions	$5\frac{1}{4}$	Two numbers are multiplied to give $5\frac{1}{4}$ . What might the numbers be?
5	Rounding	15 Seconds	Ravi ran 20 minutes about 25 seconds a particular distance in a park. What might the numbers on the stop-watch have been?

## PROCESS 2: ADAPTING A STANDARD QUESTION

We can create open-ended questions through the other one method adapting a standard question.

This is also a three-step process as follows:

Steps	Description	Example
Step 1	Identify a topic.	Measuring length using informal units
Step 2	Think a standard question	What is the length of your table measured in hand spans?
Step 3	Adapt it to make a good question	Can you find an object that is two hand spans long?

Some more examples are given below to create open-ended questions through the method adapting a standard question:

Example	Step 1	Step 2	Step 3
	Identify a topic	Think a standard question	Adapt it to make a good question
1	Money Counting	What are the different coins available?	How many different coins can you get to make Rs 50?
2	Shape	What is a circle?	How many things can you write about this circle?
3	Addition	$568 + 259 =$	$\begin{array}{r} 5 \blacksquare 8 \\ \blacksquare \blacksquare 9 \\ \hline 8 2 \blacksquare \end{array}$
4	Subtraction	$826 - 157 =$	Change the digits in its places so that the difference is between 50 and 100.
5	Time	What is your wake up time?	Mention the time for your daily activities.

### 6. EDUCATIONAL IMPLICATIONS:

- Open-ended questions develop more than remembering a fact or reproducing a skill.
- Students can learn by answering the questions and the teacher learns about each student from the attempt.
- It develops creativity due to there exist several acceptable answers.
- The students can become aware of where their understanding is incomplete.
- The students gain a better understanding of the concepts involved.
- It is possible to make up own 'good' questions for any topic and any year level.

## 7. CONCLUSION:

The open-ended type questions are not posing in the teaching of Mathematics either in classroom or in examinations. It is need to develop higher order thinking through open-ended questions. First of all need to recognize the importance of open-ended questions and implement not only in the classroom but also in examinations. Then only possible to develop and enhance the ability of the higher order thinking concerned analysis, creating, synthesis, critical thinking etc.

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# **STATE LEVEL SEMINAR ON “MATHEMATICS TEACHING FOR EMERGING INDIA”**

On 21<sup>ST</sup> December,2019.

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Sub-Theme : Handling Abstractions in Mathematics.

Title of the Topic: Addition and subtraction formula in fractions.

## Introduction:-

ఒకప్పుడు నేను విద్యార్థిగా ఉన్నప్పుడు భిన్నాలు నేర్చుకొనే సందర్భములో నాకు అవరోధం కలిగించిన అంశాలు నేటి విద్యార్థులలో 70% విద్యార్థులకు కూడ నేర్చుకోవడంలో అవరోధం కలిగిస్తున్నాయని, వారి పరీక్షలలో మూల్యాంకనం చేస్తున్న సందర్భంలో గత 15 years నుండి గమనిస్తున్నాను.

Example:

$$\frac{1}{2} + \frac{1}{3} = \frac{2}{5}$$

(మొత్తం భాగాలు 5, select చేసుకున్న భాగాలు 1+1=2 అని అనుకోవడం)

నేను attend అయ్యే గణిత కాంప్లెక్స్ సమావేశంలో కూడ ప్రతి teacher ఇదే అంశాన్ని ప్రస్తావిస్తూ వున్నారు. ఒక్కొక్కరు విద్యార్థుల స్థాయిని బట్టి వివిధ రకాలుగా బోధిస్తున్నామని తెలియజేయడం జరిగింది. కానీ పెద్ద సంఖ్యలు గల సమస్యలు వచ్చినప్పుడు తాము నేర్చుకున్న ఊహాజనిత పద్ధతిలో (క.సా.గు. ను హారంతో భాగించుకొని లవంతో గుణించడం) చెబుతున్నామని, కానీ విద్యార్థులు చాలా మంది ఈ ఊహాజనిత పద్ధతిని మరచిపోవడం వలన గణితం పై ఆసక్తిని కోల్పోతున్నారని చెప్పడం పై అందరం ఏకాభిప్రాయానికి రావడం జరిగింది.

నేను సూచించిన ఈ క్రింది ఇవ్వబడిన పద్ధతిలో విద్యార్థులు సమస్యలను సులభంగా సాధిస్తుండటంతో ఈ Innovative formula ని SCERT వారి దృష్టికి ఈ సెమినార్ ద్వారా తీసుకొని రావాలనే ఉద్దేశ్యంతో ఇందులో పాల్గొంటున్నాను.



## Objectives

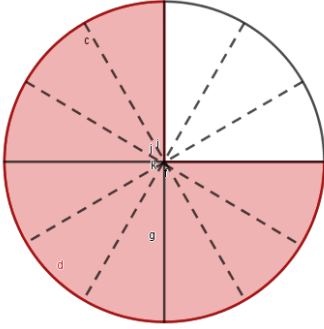
- విజాతి భిన్నాలను కూడడం, తీసివేయడం లాంటి సమస్యల సాధనలో హారాల యొక్క క.సా.గు ఎందుకు కనుగొంటారో తెలుసుకొంటారు.
- హారాలను ఎందుకు కూడగూడదో, తీసివేయకూడదో తెలుసుకుంటాడు.
- అపక్రమ భిన్నలలో లవము, హారంతో సమానంగా మరియు ఎక్కువ వున్నదానిపై అవగాహన పొందుతారు..
- అపక్రమ భిన్నం విలువ ఏయే సంఖ్యల మధ్య వుంటుందో తెలుసుకుంటారు..
- భిన్నాలకు, భాగాహారానికి మధ్య సంబంధం తెలుసుకుంటారు..
- సున్నును ఏదేని సంఖ్యచే భాగించిన దాని ఫలితం సున్నా వస్తుందన్న దానిపై అవగాహన పొందుతారు..
- సూత్ర వినియోగాన్ని బీజగణితం లో కూడ వినియోగించనున్నారు.
- ఉపాధ్యాయులు అనధికారికంగా ఉపయోగిస్తున్న ఊహాజనిత పద్ధతి పై అవగాహన పొంది ఎలాంటి సమస్యనైనా వేగంగా సాధిస్తారు.

## Design and description of the innovation:

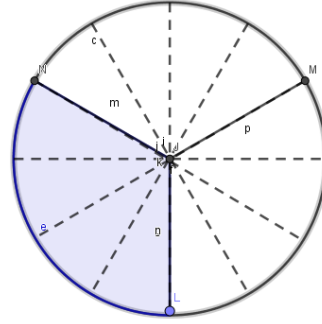
కూడికలు:-

Example  $\frac{3}{4} + \frac{1}{3} = ?$  (I fraction) I F  $\longrightarrow \frac{3}{4}$ ,

(II fraction) II F  $\longrightarrow \frac{1}{3}$ .



పటం 1.



పటం 2

$$4 \times 1 = 4$$

$$4 \times 2 = 8$$

$$4 \times 3 = 12$$

$$4 \times 4 = 16$$

$$3 \times 1 = 3$$

$$3 \times 2 = 6$$

$$3 \times 3 = 9$$

$$3 \times 4 = 12$$

పైన పేర్కొన్న దానిని గమనించినట్లు అయితే మొదటి పటంలో మొత్తం

భాగాలు 4, రెండవ పటం లో మొత్తం భాగాలు 3. రెండు పటాలలో మొత్తం

భాగాలు సమానం కాదు. కాని సంకలనం చేయాలంటే మనకు హారంలో

సమాన భాగాలు కావాలి.

దాని కొరకు మొదటి పటంలో ప్రతి భాగాన్ని మరి మూడు భాగాలుగా

విభజిస్తాము. అంటే మొత్తంగా 12 భాగాలుగా విభజిస్తాము ( $4 \times 3 = 12$ ).

అదే విధంగా రెండవ పటంలో ప్రతి భాగాన్ని మరి 4 భాగాలుగా విభజిస్తాము. దీనిలో కుడా మొత్తం భాగాలు 12 అవుతాయి. (3 X 4=12).

ఈ 12 అనేది 3,4 ల యొక్క కనిష్ట సామాన్య గుణిజం అవుతుంది కాబట్టి, భిన్నాల యొక్క హారాలను సమానం చేయాలంటే హారల యొక్క క.సా.గు. ని కనుగొనాలి.

( రెండింటిని సమాన భాగాలు చేసిన తరువాత )

పటం 1 :- 12 లో  $\frac{3}{4}$  [క.సా.గు. లో 3/4 ]

(12 లో  $\frac{3}{4}$  అంటే 12 పూర్ణాంకలను నాలుగు సమాన భాగాలను చేసి దానిలో 3 భాగాల విలువను తీసుకోవడం. అది 9 అవుతుంది. కావున 12 లో  $\frac{3}{4}$  వ భాగం 9 కి సమానం).

$$12 \times \frac{3}{4} = 9$$

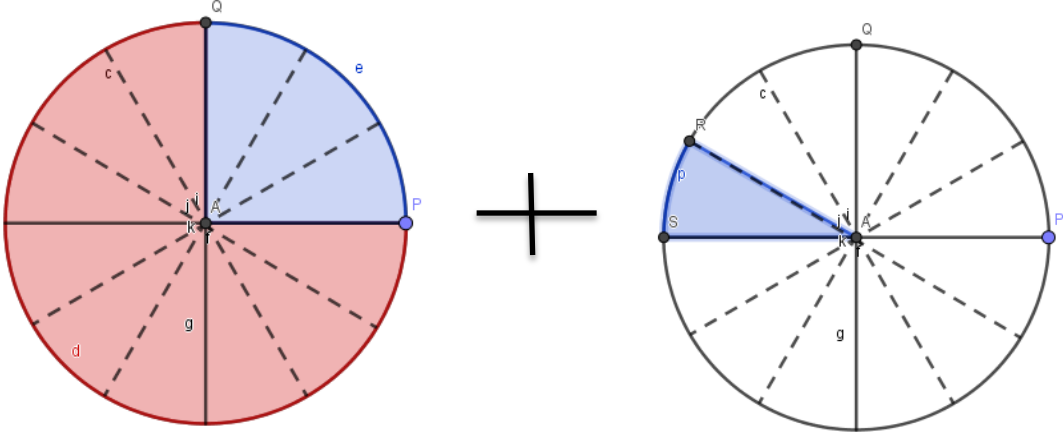
పటం 2 :- 12 లో  $\frac{1}{3}$  [క.సా.గు. లో 1/3 ]

(12 లో  $\frac{1}{3}$  అంటే 12 పూర్ణాంకాలను మూడు సమాన భాగాలను చేసి దానిలో 1 భాగం విలువను తీసుకోవడం. అది 4 అవుతుంది. కావున 12 లో  $\frac{1}{3}$  వ భాగం 4 కి సమానం)

$$12 \times \frac{1}{3} = 4$$

Select చేయబడిన మొత్తం భాగాలు 9 + 4 = 13

[ 13 భాగాలను re-arrange చెయ్యగా ]



$$= \frac{12}{12} + \frac{1}{12} = 1 + \frac{1}{12} = 1 \frac{1}{12}$$

$1 \frac{1}{12}$  లో ఒక పూర్ణాంకం(1) మరియు ఒక క్రమ భిన్నం( $\frac{1}{12}$ ) వుంది. ఈ

రెండింటి కలయికను మిశ్రమ భిన్నంగా చెప్పవచ్చును .

( $1 \frac{1}{12}$  ను భాగహారం తో పోల్చినప్పుడు పూర్ణాంకం (1) భాగపలాన్ని

సూచిస్తే, లవం లో వున్న 1 శేషాన్ని మరియు హారంలో వున్న 12

విభాజకంను సూచిస్తుంది).

మనం  $1 \frac{1}{12}$  ను లవ,హార రూపంలో సూచించాలంటే మీరు 5 వ

తరగతిలో భాగహారం ఎలా చేయాలో, సరిచుడాలో నేర్చుకున్నారు. అందులో

భాగపలాన్ని విభాజకం తో గుణించి, వచ్చిన పలితానికి శేషంను కూడిన

విభాజ్యం వస్తుంది. అలా వచ్చిన విభాజ్యంను మనం లవంగాను, విభాజకం ను

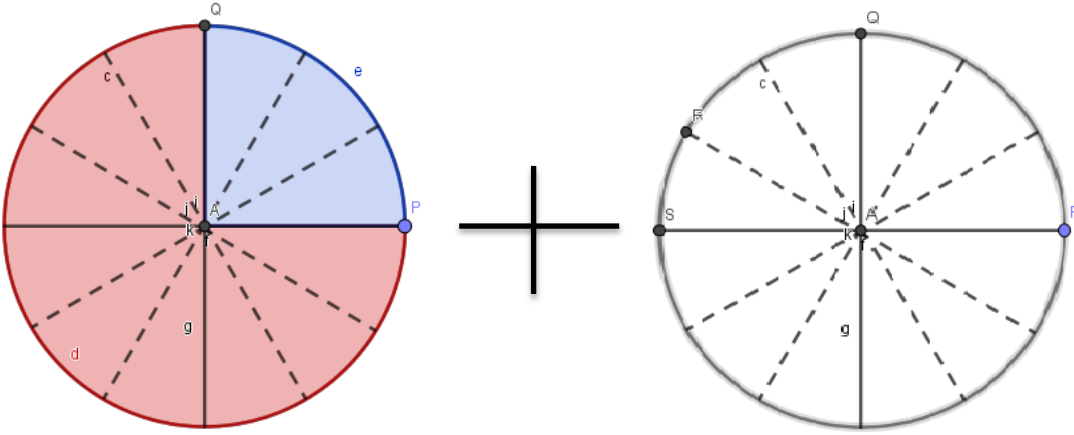
హారంగాను తీసుకోవాలి.

$$\text{అప్పుడు } 1 \frac{1}{12} = \frac{(12 \times 1) + 1}{12} = \frac{13}{12} \text{ అవుతుంది.}$$

దీనినే అపక్రమ భిన్నం అంటారు. ఇందులో లవం , హారం కంటే

పెద్దదిగా వుంటుంది. దీని విలువ 1 కంటే ఎక్కువగా వుంటుంది.

ఒకవేళ select చేయబడిన భాగాలు 12 అయితే ఏమి అవుతుంది ?



$$= \frac{12}{12} + \frac{0}{12} = \frac{12}{12} + 0 = \frac{12}{12} = 1 \text{ [ "0" ను ఏదేని సంఖ్యచే భాగించిన విలువ "0" వస్తుంది. ]}$$

ఈ విధంగా  $\frac{12}{12}$  భిన్నాన్ని కూడ అపక్రమ భిన్నం అంటారు. ఇందులో లవం, హారంతో సమానంగా వుంటుంది. దీని విలువ 1 అవుతుంది.

అంటే అపక్రమ భిన్నం విలువ 1 లేదా అంతకంటే ఎక్కువగా వుంటుంది.

$$\frac{3}{4} + \frac{1}{3} \text{ సమస్య సాధన సంక్షిప్తంగా}$$

$$= \frac{(12 \times \frac{3}{4}) + (12 \times \frac{1}{3})}{12}$$

$$= \frac{(12 \times \frac{3}{4}) + (12 \times \frac{1}{3})}{12} = \frac{(\text{క.సా.గు X I F}) + (\text{క.సా.గు.X II F})}{\text{క.సా.గు}}$$

$$= \frac{(3 \times 3) + (4 \times 1)}{12}$$

$$= \frac{9+4}{12} = \frac{13}{12} \quad [\text{క.సా.గు. ను హారంతో భాగించి మరియు లవంతో}$$

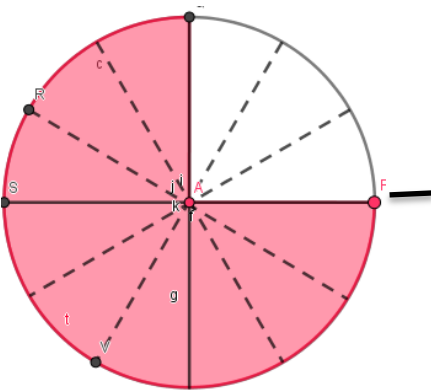
గుణించాలి.]

### తీసివేతలు:-

రెండు విజాతి భిన్నాల తీసివేత ప్రక్రియలో కూడా హారంలో వున్న మొత్తం భాగాలను సమానం చేయాలి. అందుకోసం మనం సంకలనము లో చేసిన విధంగా హారాల క.సా.గు. కనుగొనాలి.

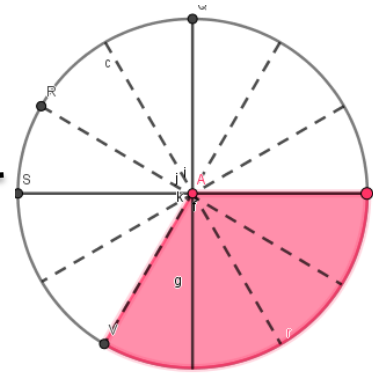
$$\text{ఉదా: } \frac{3}{4} - \frac{1}{3}$$

పటం. 1



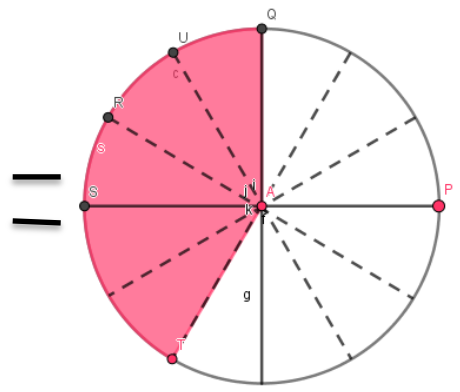
$$\frac{9}{12}$$

పటం. 2



$$\frac{4}{12}$$

పటం. 3



$$\frac{5}{12}$$

సంకలనం లో మాదిరిగా తీసివేతలో కూడా మొదటి పటంలో 12 లో  $\frac{3}{4}$

అంటే 9 భాగాలు తీసుకోవాలి. రెండవ పటంలో 12 లో  $\frac{1}{3}$  అంటే 4 భాగాలు తీసుకోవాలి.

9 భాగాలనుండి 4 భాగాలను తీసివేసినచో మూడవ పటంలో మాదిరిగా 5 భాగాలు మిగులుతాయి.

$$\begin{aligned} &= \frac{(\text{క.సా.గు X I F}) - (\text{క.సా.గు. X II F})}{\text{క.సా.గు}} \\ &= \frac{(12 \times \frac{3}{4}) - (12 \times \frac{1}{3})}{12} = \frac{9-4}{12} = \frac{5}{12} \end{aligned}$$

**బీజగణితం లో ఉపయోగం:-**

$$\begin{aligned} \text{Example: } & \frac{2x^2}{3} + \frac{4x^2}{15y} \\ &= \frac{(15y \times (\frac{2x^2}{3})) + (15y \times (\frac{4x^2}{15y}))}{15y} \\ &= \frac{(5y \times 2x^2) + (1 \times 4x^2)}{15y} = \frac{10x^2y + 4x^2}{15y} \end{aligned}$$

## Outcome of the innovation:-

$$\text{Addition formula} = \frac{(\text{క.సా.గు X I F}) + (\text{క.సా.గు.X II F})}{\text{క.సా.గు}}$$

$$\text{Subtraction formula} = \frac{(\text{క.సా.గు X I F}) - (\text{క.సా.గు.X II F})}{\text{క.సా.గు}}$$

కృతజ్ఞతలు.



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## **THEME: “MATHEMATICS TEACHING FOR EMERGING INDIA”**

### **Sub Theme : Handling abstractions in Mathematics Teaching**

#### **Title of the Topic :**

**Handling abstractions in UP Level and Secondary Level Mathematics by using VEDIC MATHEMATICS SUTRAS**

#### **Introduction :**

The Vedic Sutras apply to and cover almost every branch of Mathematics. They apply even to complex problems involving a large number of mathematical operations. Application of the Sutras saves a lot of time and effort in solving the problems, compared to the formal methods presently in vogue. Though the solutions appear like magic, the application of the Sutras is perfectly logical and rational. The computation made on the computers follows, in a way, the principles underlying the Sutras. The Sutras provide not only methods of calculation, but also ways of thinking for their application. The solving of algebraic solutions using Vedic Mathematics seeks to present an integrated approach to learning Mathematics with keenness of observation and inquisitiveness ,avoiding the monotony of accepting theories and working from them mechanically. The explanations offered make the processes clear to the learners. The logical proof of the Sutras is detailed in algebra, which eliminates the misconception that the Sutras are a jugglery.

#### **Objectives:**

- Multiplication of Binomials and Polynomials by using Urdhva Thiryagbhyam Sutra.
- Finding the solution of Pair of linear Equations in two variables by using Paravarthya Yojayet sutra.
- Finding the solution of Simple Equations by using Sunya Samyam Samuchhaye Sutra.
- Finding the solution of Linear Equations in two variables by using Anurupye Sunyamanyath sutra.

#### **Design of innovation:**

- ❖ It is a creative procedure by using VEDIC SUTRAS and it is good form and better function easily understood abstract thinking of Mathematics to a student.

❖ It is an illustrated method by using **Vedic Sutras**.

**Description of innovation:**

It implements new idea to find the product of binomials and polynomials in different order. These ideas creates to conceive of new ideas for finding solutions of two variable linear equations.

**Outcome of the innovation:**

- The students find out the product of binomials and polynomials.
- The students solve the Pair of Linear Equations in two variables
- The students solve the different quadratic equations '

**Finding the Product of Binomials, Polynomials by using Urdhva Thiryagbhyam( 3<sup>rd</sup> Sutra)**

The meaning of Urdhva Thiryagbhyam is Vertically and Cross Wise

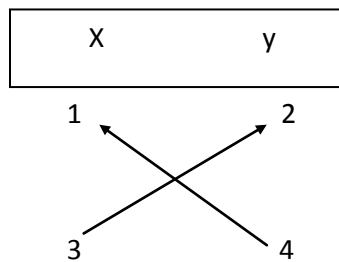
**1. Multiplication of binomials.**

$$(x+2y)(3x+4y)$$

‘ x ’ coefficient 1 and ‘ y ’ coefficient 2

$$(3x+4y)$$

‘ x ’ coefficient 3 and ‘ y ’ coefficient 4



$$1 \times 3 / (3 \times 2 + 4 \times 1) / 2 \times 4$$

using cross wise method from Sutra

$$3 / (6+4) / 8$$

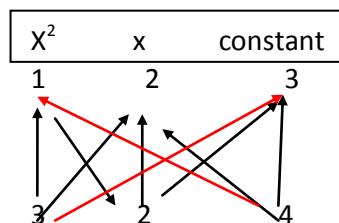
$$3 / 10 / 8$$

The product of binomials  $3x^2 + 10xy + 8y^2$

**2. Multiplication of Polynomials**

For example

$$x^2 + 2x + 3, \quad 3x^2 + 2x + 4$$



$$1 \times 3 / (1 \times 2 + 3 \times 2) / (3 \times 3 + 2 \times 2 + 1 \times 4) / (2 \times 4 + 3 \times 2) / 3 \times 4$$

$$3 / 8 / (17) / 14 / 12$$

Therefore the product is  $3x^4 + 8x^3 + 17x^2 + 14x + 12$

## Finding the solution for pair of Linear Equations in two variables by using Paravarthya Yojayet (4<sup>th</sup> sutra)

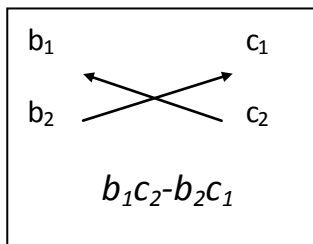
The meaning of Paravarthya Yojayet is Transpose and apply or adjust.

For example find the solution of pair of linear equations

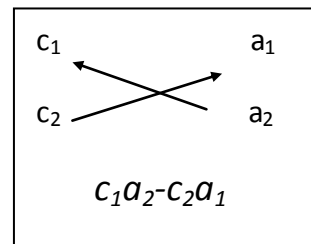
$$a_1x + b_1y = c_1 \text{ and } a_2x + b_2y = c_2$$

Find the numerator of 'x' and 'y' by transpose method in the following way using Vedic sutra

numerator of 'x' by transpose method

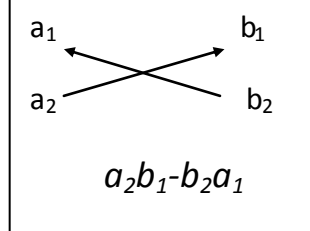


numerator of 'y' by transpose method



Take the coefficients in cyclic order only

Denominator of 'x' and 'y' by Transpose sutra



Here in the above we should start cross multiply from  $a_2$

Now the solution of equations

$$x = \frac{\text{numerator of 'x'}}{\text{denominator of 'x'}}$$

$$y = \frac{\text{numerator of 'y'}}{\text{denominator of 'y'}}$$

$$x = \frac{b_1c_2 - b_2c_1}{a_2b_1 - b_2a_1}$$

$$y = \frac{c_1a_2 - c_2a_1}{a_2b_1 - b_2a_1}$$

**this method we can use for any numerical values of  $a_1, a_2, b_1, b_2, c_1, c_2$**

**for example the solution of  $2x + 3y = 7$  and  $3x + 7y = 13$  is  $x = 2$  and  $y = 1$**

## Finding the solutions for Simple Equations by using Sunyam samya samuchhaye (5<sup>th</sup> sutra)

The Sutra 'Sunyam Samyasamuchhaye' says the 'Samuhhaya is the same, that Samuchhaya is Zero.' i.e., it should be equated to zero.

The term 'Samuchhaya' has several meanings under different contexts.

### **Case:1**

We interpret, ' Samuchhaya ' as a term which occurs as a common factor in all the terms concerned and proceed as follows.

**Example:**  $10x+6x-3x-2x-11x = 0$

In the above equation common factor is 'x' then the solution is '0'

**Case:2** We interpret 'Samuchhaya' as product of independent terms in expressions like (x+a) (x+b)

**Example:**  $(x+2)(x+12) = (x+6)(x+4)$

In the above equation 2 , 12 and 6 , 4 are independent terms.

Their product is  $2 \times 12 = 24$  and  $6 \times 4 = 24$

If the product of independent terms in LHS = Product of independent terms in RHS

Its solution is zero.

### **Case:3**

We interpret ' Samuchhaya ' as the sum of the denominators of two fractions having the same numerical numerator.

**Example:**

$$\frac{1}{3x-1} + \frac{1}{4x-1} = 0$$

When the numerators are same and the sum of the denominators equal to 'zero'

**From the above equation numerators are same and the sum of the denominators is equal to zero**

$$3x-1+4x-1 = 0$$

$$7x-2 = 0$$

$$7x = 2$$

$$x = \frac{2}{7}$$

$$\text{solution is } x = \frac{2}{7}$$

### **Case:4**

We now interpret ' Samuchhaya ' as combination or total.

If the sum of the numerators and the sum of the denominators be the same, then that sum = 0.

**For example**

$$\frac{3x+5}{3x+4} = \frac{3x+4}{3x+5}$$

$$\text{Sum of the numerators} = 3x+5+3x+4 = 6x+9$$

Sum of the denominators =  $3x+4+3x+5 = 6x+9$

The solution is  $6x+9 = 0$

$$6x = -9$$

$$x = \frac{-9}{6}$$

$$x = \frac{-3}{2}$$

#### Case:4

**Samuchhaya means the combination :** In the algebraic total if any common numerical factor then remove that factor and equated sum to zero

**For example**

$$\frac{3x + 4}{6x + 5} = \frac{x + 1}{2x + 5}$$

Sum of the numerators =  $3x+4+x+1 = 4x+5$

Sum of the denominators =  $6x+5+2x+5 = 8x+10$

$$= 2(4x+5)$$

Remove the numerical factor and equated to zero

$$4x+5 = 0$$

$$4x = -5$$

$$x = \frac{-5}{4}$$

#### Special case for Quadratic Equation :

'Samuchhaya' with the same meaning as above, i.e., **case: 4**, we solve the problems leading to quadratic equations. In this context, we take the problems as follows;

If  $N1 + N2 = D1 + D2$  and also the differences

$N1 \sim D1 = N2 \sim D2$  then both the things are equated to zero, the solution gives the two values for x.

For example

$$\frac{3x + 4}{4x + 7} = \frac{5x + 6}{4x + 3}$$

#### Step1:

Sum of the numerators =  $3x+4+5x+6 = 8x+10$

Sum of the denominators =  $4x+7+4x+3 = 8x+10$

The sum is equal to zero =  $8x+10=0$

$$x = \frac{-5}{4}$$

#### Step2:

Observe the coefficients of 'x<sup>2</sup>' on cross multiplication  
'12' and '20'

Difference of numerator and denominator =  $x+3$  (LHS)

Difference of numerator and denominator =  $x+3$  (RHS)

Then the solution is  $x+3=0$

$$x = -3$$

#### Case:5

Samuchhaya means 'Total'

According to sutra the sum of denominators on LHS and the sum of denominators on RHS are same ,then that sum or total is equal to 'zero'.

**For example:**

$$\frac{1}{x-5} + \frac{1}{x-11} = \frac{1}{x-6} + \frac{1}{x-10}$$

Sum of the denominators of LHS =  $x-5+x-11=2x-16$

Sum of the denominators of RHS =  $x-6+x-10=2x-16$

$$2x-16=0$$

Solution is  $x = 8$

### **Finding the solutions of Linear Equations in two variable by using Anurupye sunyamanyath (6<sup>th</sup> sutra)**

**Anurupye Sunyamanyath means If one in the ratio the other will be equal to zero.**

The ratio of the coefficient 'x' and constant terms of the both equations is same. Then substitute 'y' is equal to 'zero' to get 'x' value.

The ratio of the coefficient 'y' and constant terms of the both equations is same. Then substitute 'x' is equal to 'zero' to get 'y' value.

For example: To find the solution of

$$5x+8y=40 \text{ and } 10x+11y=80$$

$$\text{The ratio of the 'x' coefficients} = \frac{5}{10} = \frac{1}{2}$$

$$\text{and constants is} = \frac{40}{80} = \frac{1}{2}$$

the ratio of 'x' coefficients and constants is equal therefore substitute  $y=0$  to get 'x'

$$\text{substitute } y=0 \text{ in one equation } 5x+8y = 40$$

$$5x+8(0) = 40$$

$$5x = 40$$

$$x = 8$$

therefore the solution is  $x=8$  and  $y=0$

### **Conclusion :**

- Urdhva Thiryagbhyam Sutra used to find the product of binomials and polynomials
- Paravarthya Yojayeth sutra used to find the solution of linear equations in two variables.
- Sunya Samya Samuchhaye sutra used to find the solutions of simple equations and special case quadratic equations.
- Anurupye Sunyamanyath sutra is also used to find the solution linear equations in two variables in a particular case.
- All the above methods used by Vedic sutras are other than conventional methods.

### **References :**

(i) VEDIC MATHEMATICS

(ii) Topics selected from VII, IX and X class Mathematics Text books of SCERT Telangana State and developed innovative ideas for solutions from Vedic Sutras and activities other than text book activities.

**PAPER PRESENTATION ON THE OCCASION OF  
NATIONAL MATHEMATICS DAY**

**TO BE HELD ON 21/12/2019 AT SCERT, HYDERABAD, TELANGANA**

**PERSONAL DETAILS :**

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**1. Sub - Theme :**

**Handling Abstraction in mathematics**

**Title of the Topic :** Fibonacci Gauge పరికరం ద్వారా కొలిచి చూడడం

**Objectives:**

Fibonacci Gauge కనుక్కోనేందుకు ఉపయోగించే ఈ పరికరం ద్వారా 8వ తరగతి విద్యార్థులకు ఆసక్తి పెంపొందేలా చేయడం మరియు గణితంలోని అందాలను చూడగలుగుతారు. విద్యార్థులు జ్యామితీయ భావనలను చేయడం నేర్చుకుంటారు. మరియు ఈ దివ్య అనుపాత నిష్పత్తిని మానవ శరీర భాగాలతో పోల్చి చూస్తారు. ఈ నిష్పత్తిని ప్రయోగ పూర్వకంగా చేసి వివిధ ప్రాజెక్ట్లలో అనవదించి చేయడం జరుగును.

**Back Ground of the Topic :**

ఈ గోల్డెన్ రేషియో ని మొదటగా గ్రీకు శాస్త్రవేత్త అయినా “ ఫైడియాస్ ” క్రీ. పూ. 3000 సం॥రాలకే చెప్పారు. మళ్ళీ “ ఆచార్య హేమాచంద్ర ” అనే భారతీయ శాస్త్రవేత్త క్రీ. శ. 1000 లో తెలిపారు. వీరు తెలిపిన గోల్డెన్ రేషియో గురించి ప్రాచుర్యంలోకి రాలేదు.

I

మధ్య యుగానికి చెందిన **Fibonacci** శాస్త్రవేత్త ఒక అంకేల క్రమమును కనిపెట్టడం జరిగింది. ఇది ప్రాచూర్యంలోకి వచ్చింది. దీనికే **Fibonacci sequence** అనే పేరు వచ్చింది. అది 0,1,2,3,5,8,13,21,34,55,89,144,.....

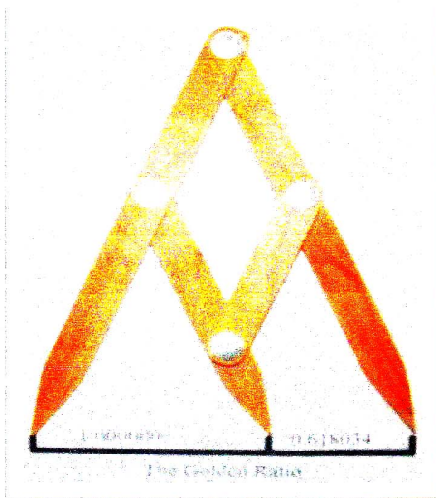
ఏ సంఖ్యనైనా ముందు సంఖ్యను కలపడం ద్వారా వస్తుంది. ఈ సీక్వెన్స్ లో ఉన్న ఏ వరుస సంఖ్యల నిష్పత్తిని అయినా 1.618 కి సమానంగా వస్తుంది.

ఈ సీక్వెన్స్ మనకు ప్రకృతిలో ఉన్న అన్నింటిలో కనబడును. ఉదా॥ చెట్లు వాటి కొమ్మలను ఏర్పరుచుకోవడంలో, సత్తగుల్ల, చెట్లు వాటి ఆకులను ఏర్పరుచుకోవడంలో, విశ్వంలో ఉన్న గెలాక్సీ ఏర్పడిన క్రమం, స్టార్ ఫిష్ కూడా ఈ నిష్పత్తిలో ఉంటుంది.

మరియు పువ్వులు వాటి రెక్కల క్రమంలో, గులాబీ పువ్వు ఏర్పడిన క్రమంలో, ఫైనాపీల్ లో కణుతులు, కాలిఫ్లవర్ లో, అరటి పండు ఏర్పడిన క్రమంలో కూడా గోల్డెన్ రేషియో ఉంటుంది.

## Designing of Innovation :

ఫిభోనాకి సీక్వెన్స్ లో ఉన్న ఏవేని 3 వరుస సంఖ్యల పొడవులను అట్ట ముక్కలు / చెక్క ముక్కలను తీసుకోని ప్రక్క పటంలో చూపినవిధంగా కట్ చేసి నట్లతో/ స్ట్రూలతో బిగించవలెను. ఇది ఒక విభాగిని ఆకారంలో ఉండును.



## Description of the innovation

**Fibonacci Gauge** ఉపయోగించి క్రమ పంచభుజిలో ఈ నిష్పత్తిని ఉండేల చూసుకోవడం, వివిధ గణిత సమస్య నిర్మాణాలు గీయడంలో, తాజ్ మహల్ పటంలో, పార్థెనాన్ దేవాలయ పటంలో గల గోల్డెన్ రేషియో కనుక్కోవడంలో నైపుణ్యాన్ని సాధిస్తాడు.

విద్యార్థులు మానసిక, చలనత్మాక రంగానికి చెందిన నైపుణ్యాలను పొందుతారు.



## Out come of the Innovation:

విద్యార్థులు అమూర్తభావన నుండి మూర్త భావనలో గోల్డెన్ రేషియోను కనుక్కోవడం జరుగును. దీనితో సమస్య సాధన నైపుణ్యం, నిర్ణయం తీసుకోనే నైపుణ్యం పెంపొందును.

గణితాన్ని విద్యార్థులు పరిమాణాలు, ఆకారాలు, రూపాలు, నమూనా సన్నివేశాల ద్వారా ఉపయోగించవచ్చు.

బ్లామ్స్ వర్గీకరణలో 1. జ్ఞాపకంలో ఉంచడం, 2. అవగాహన చేసుకోవడం, 3. వినియోగించుట, 4. విశ్లేషణ చేయుట, 5. మూల్యంకన చేయుట, 6. సృష్టించుట, ఉత్పత్తి చేయడం జరుగును.

దీనిలో భాగంగా ఈఫీల్ టవర్ మొత్తం 1024 ft ఎత్తు ఉంటుంది.

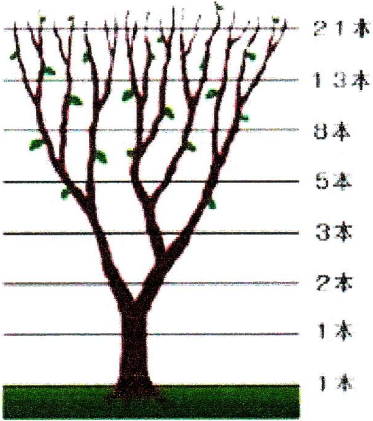
దీనిలో ఐదు భాగాల ఎత్తులు వరుసుగా

186 ft , 380 fts, 644 fts, 906 fts, 1024 fts గా ఉంటాయి .

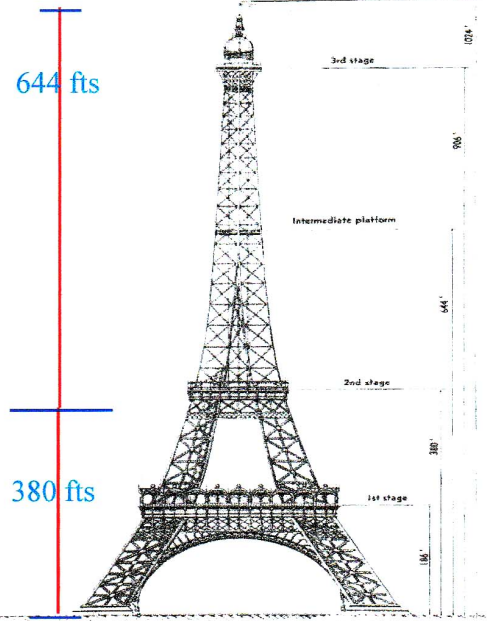
ఈ ఎత్తుల యొక్క నిష్పత్తి గోల్డెన్ రేషియోకి

దగ్గరగా ఉంటుందని విద్యార్థి తెలుసుకుంటాడు.

$$644 \text{ fts} : 380 \text{ fts} = 1.694 : 1$$



అలాగే చెట్లు వాటి కొమ్మలను ఏర్పరుచుకునే క్రమం కూడ ఫిబోనాకీ అంకెల క్రమంలో ఉండునని విద్యార్థి గుర్తిస్తాడు.



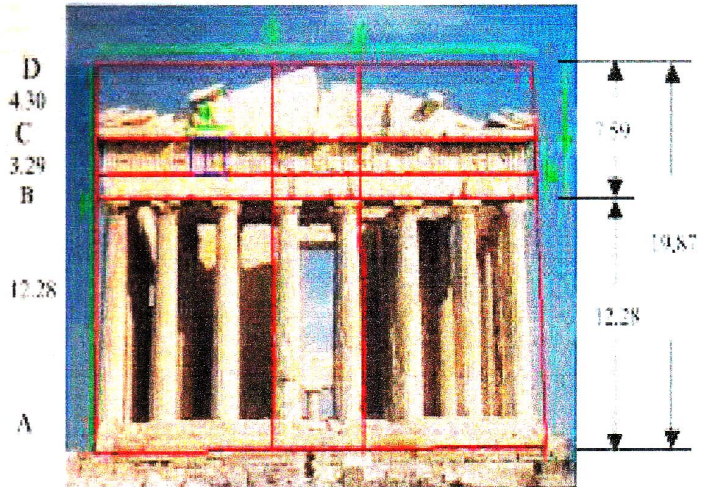
క్రి|| పూ|| 5వ శతబ్దిలో గ్రీసులో నిర్మించిన ప్రఖ్యత పార్థెనాన్

దేవలయం కూడా గోల్డెన్ రేషియోలో నిర్మించడం జరిగింది.

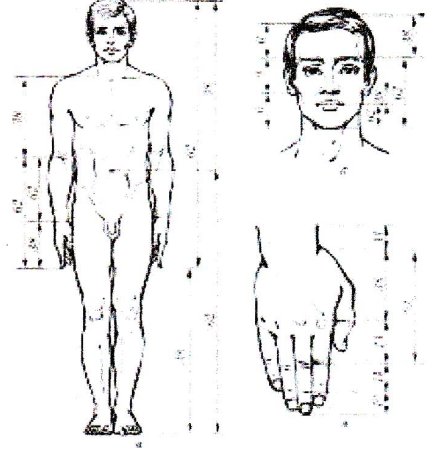
దీని ఎత్తు యొక్క నిష్పత్తి గోల్డెన్ రేషియోకి దగ్గరగా ఉంటుంది.

$$19.87 : 12.288 = 1.618 : 1$$

$$12.28 : 7.59 = 1.617 : 1$$



మానవుని శరీరం గోల్డెన్ రేషియో లో ఉంటుందని విద్యార్థి స్వయంగా క్రింది విధంగా తెలుసుకుంటాడు.



1. ఎత్తు : నాభి నుండి పాదాగ్రం వరకు గల పొడవు.
2. భుజరేఖ పొడవు : తల పొడవు
3. చేతి వేళ్ళ చివరి నుండి మోచేతికి దూరం : మణికట్టు నుండి మోచేతికి దూరం
4. నాభి నుండి మోకాళ్ళ వరకు దూరం : మోకాళ్ళు నుండి పాదంకు దూరం

విద్యార్థులు అనుపాత నిష్పత్తులను ఆసక్తిగా నేర్చుకుంటారు. నిత్యజీవితంలో మైనపు బోమ్మలు, జంతువుల విగ్రహాలు తయారీలో ఈ నిష్పత్తిని వినియోగిస్తారు.

**Implications** గోల్డెన్ రేషియో విద్యార్థులు నేర్చుకోవడం వల్ల పై చదువులు చదివేటప్పుడు అనగా ఇంజనీరింగ్ ప్లాన్ గీసేదానిలో ఉపయోగిస్తారు. విద్యార్థి తాను స్వయంగా ఇలాంటి పరికరాలను ఉపయోగించి గోల్డెన్ రేషియో ఉందో లేదో కనుక్కుంటాడు. గణితం అంటే భయం ఉన్న విద్యార్థులు కూడా స్వయంగా ఇటువంటి కృత్యాల ద్వారా నేర్చుకొవడం వల్ల గణితం పట్ల ఆసక్తి పెరిగి విద్యార్థులు గణితం వైపు మొగ్గు చూపుతారు.

భవిష్యత్తులో గణితం తీసుకోవడానికి అవకాశం ఎక్కువగా ఉంటుంది.

**References :** 8వ తరగతి పాఠ్యపుస్తకం, ఇంటర్నెట్

## ABSTRACTION IN MATHEMATICS AND MATHEMATICS LEARNING

KANDAGATLA RAKESH KUMAR\*

### Abstract

*The aim of this paper is to contrast abstraction in mathematics with empirical abstraction in mathematics learning. In particular, we want to clarify “the relation between mathematical objects and thinking processes” A discussion of the relationship between abstract-apart objects and abstract-general concepts leads to the conclusion that a key component in learning about fundamental mathematical objects is the formalisation of empirical concepts. Historically, some more advanced mathematical objects have been constructed by a process similar to empirical abstraction*

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## ABSTRACTION IN MATHEMATICS AND MATHEMATICS LEARNING

### Abstract

The aim of this paper is to contrast abstraction in mathematics with empirical abstraction in mathematics learning. In particular, we want to clarify “the relation between mathematical objects and thinking processes” A discussion of the relationship between abstract-apart objects and abstract-general concepts leads to the conclusion that a key component in learning about fundamental mathematical objects is the formalisation of empirical concepts. Historically, some more advanced mathematical objects have been constructed by a process similar to empirical abstraction.

### Introduction

It is claimed that, since mathematics is essentially a self-contained system, mathematical objects may best be described as abstract-apart. On the other hand, fundamental mathematical ideas are closely related to the real world and their learning involves empirical concepts. These concepts may be called abstract-general because they embody general properties of the real world. A discussion of the relationship between abstract-apart objects and abstract-general concepts leads to the conclusion that a key component in learning about fundamental mathematical objects is the formalisation of empirical concepts. A model of the relationship between mathematics and mathematics learning is presented which also includes more advanced mathematical objects.

### ABSTRACTION IN MATHEMATICS

What does it mean to say that mathematics is “abstract”? Mathematics is a self-contained system separated from the physical and social world:

Mathematics uses everyday words, but their meaning is defined precisely in  $\forall$ ! relation to other mathematical terms and not by their everyday meaning. Even the syntax of mathematical argument is different from the syntax of everyday language and is again quite precisely defined.

Mathematics contains objects which are unique to itself. For example, although everyday language occasionally uses symbols like  $x$  and  $P$ , objects like  $x \cdot 0$  and  $(-1)$  are unknown outside mathematics.

A large part of mathematics consists of rules for operating on mathematical objects  $\forall$  and relationships. Sierpinski calls these “the rules of the game”. It is important that students learn to manipulate symbols using these rules and no others.

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We claim that the essence of abstraction in mathematics is that mathematics is self-contained: An abstract mathematical object takes its meaning only from the system at all within which it is defined. Certainly abstraction in mathematics includes ignoring certain features and highlighting others, as Sierpinski levels emphasises. But it is crucial that the new objects be related to each other in a consistent system which can be operated on without reference to their previous meaning. Thus, self-containment is paramount.

Historically, mathematics has seen an increasing use of axiomatics, especially over the last two centuries. For example, numbers were initially mathematical objects based on the empirical idea of quantity. Then mathematicians such as Dedekind and Peano reconceptualised numbers in axiom systems which were independent of the idea of quantity. Euclid, Hilbert, and others performed a similar task for geometry. But, as Kleiner (1991) states, “whereas Euclid’s axioms are idealizations of a concrete physical reality ... in the modern view axioms are ... simply assumptions about the relations among the undefined terms of the axiomatic system” (p. 303). In other words, mathematics has become increasingly independent of experience, therefore more self-contained and hence more abstract.

To emphasise the special meaning of abstraction in mathematics, we shall say that mathematical objects are abstract-apart. Their meanings are defined within the world of mathematics, and they exist quite apart from any external reference.

### **So why is mathematics so useful?**

Mathematics is used in predicting and controlling real objects and events, from calculating a shopping bill to sending rockets to Mars. How can an abstract-apart science be so practically useful?

One aspect of the usefulness of mathematics is the facility with which calculations can be made: You do not need to exchange coins to calculate your shopping bill, and you can simulate a rocket journey without ever firing one. Increasingly powerful mathematical theories (not to mention the computer) have led to steady gains in efficiency and reliability.

But calculation facility would be useless if the results did not predict reality. Predictions are successful to the extent that mathematics models appropriate aspects of reality, and whether they are appropriate can be validated by experience. In fact, one can go further and claim that the mathematics we know today has been developed (in preference to any other that might be imaginable) because it does model significant aspects of reality faithfully.

How is it that the axiomatic method has been so successful in this way? The answer is, in large part, because the axioms do indeed capture meaningful and correct patterns. ... There is nothing to prevent anyone from writing down some arbitrary list of postulates and proceeding to prove theorems from them. But the chance of those theorems having any practical application [is] slim indeed.

Many fundamental mathematical objects (especially the more elementary ones, such as numbers and their operations) clearly model reality. Later developments (such as combinatorics and differential equations) are built on these fundamental ideas and so even if indirectly. Hence all mathematics has some link back to  $\exists$  also reflect reality reality.

## **EMPIRICAL ABSTRACTION IN MATHEMATICS LEARNING**

### **Learning fundamental mathematical ideas**

Students learn about many fundamental, abstract mathematical objects in school. In this section, we discuss the meaning of abstraction in this learning context. We begin by looking at some examples.

*Addition.* Between the ages of 3 and 6, most children learn that a given set of objects contains a fixed number of objects. A little later, they realise that two sets can be combined and that the number of objects in the combined set can be determined from a procedure which later becomes the operation of  $\exists$  the number of objects in each set addition. Students learn these fundamental arithmetical ideas from counting experiences: They find that repeatedly counting a given set of objects always gives the same number, no matter how often it is done and in which order. As they recognise more and more patterns, counting a combined set is gradually replaced by “counting on” and eventually the use of “number facts”

*Angles.* There is good evidence that, at the beginning of elementary school, students have already formed classes of angle situations such as corners, slopes, and turns (Mitchelmore, 1997). To acquire a general concept of angle, students need to see the similarities between them and identify their essential common features (two lines meeting at a point, with some significance to their angular deviation). Even secondary students find it difficult to identify angles in slopes and turns, where one or both arms of the angle have to be imagined or remembered.

*Rate of change.* The most fundamental idea in calculus is rate of change, leading to differentiation. A major reform movement over the last decade or so has been concerned with making this idea more meaningful by initially exploring a range of realistic rate of change situations. In this way, students build up an intuitive idea of rate of change before studying the topic abstractly. A leading US college textbook (Hughes-Hallett et al., 1994) devotes a whole introductory chapter to exploring realistic situations, and in Australia similar materials have been published for high school calculus students.

### **Characteristics of empirical abstraction**

The above examples show how fundamental mathematical ideas are based on the investigation of real world situations and the identification of their key common features. Hence, a characteristic of the learning of fundamental mathematical ideas is similarity recognition. The similarity is not in terms of superficial appearances but in for example, in counting, space, and relationships. To get  $\exists$  underlying structure below the surface often



requires a new viewpoint, as when a student imposes imaginary initial and final lines on a turning object in order to obtain an angle.

There is a leap forward when students recognise such a similarity: As students relate together situations which were previously conceived as disconnected, they become able to do things they were not able to do before. More than that, they form new ideas (such as addition, angle, and rate of change) and are incapable of reverting to their previous state of innocence. In a sense, these new ideas embody the similarities recognised. Of course, single ideas rarely evolve in isolation; for example, the idea of angle is inextricably linked to ideas such as point, line, parallel, intersection and measurement which can also be traced to similarities students recognise in their environment.

Abstracting is an activity by which we become aware of similarities ... among our experiences. Classifying means collecting together our experiences on the basis of these similarities. An abstraction is some kind of lasting change, the result of abstracting, which enables us to recognise new experiences as having the similarities of an already formed class. ... To distinguish between abstracting as an activity and abstraction as its end-product, we shall ... call the latter a concept.

Thus number, addition, angle and rate of change are all empirical concepts, and they take their place in students' learning alongside other empirical concepts such as colour, friend, and fairness.

Piaget (1977) made a distinction between abstraction on the basis of superficial characteristics of physical objects (abstraction à particular object and abstraction on the basis of relationships perceived when the learner manipulates these objects (abstraction à partir de l'action). But both are based on the child's physical and social experience, and in both similarity recognition is essential. In using the term empirical abstraction to cover both cases, we are making the distinction between abstraction on the basis of experience and what we shall call theoretical abstraction (see below).

## **EMPIRICAL ABSTRACTION AND MATHEMATICAL ABSTRACTION**

### **From empirical concept to mathematical object**

When students learn a fundamental mathematical idea in the way described above, three things happen: They learn an empirical concept, they learn about a mathematical object, and they learn about the relationship between the empirical concept and the mathematical object. Empirical concepts are often rather fuzzy and difficult to define. For example, the empirical concept of circle is that of a perfectly but "perfect roundness" can only be defined by showing examples. A round object circle becomes a mathematical object only when it is defined as the locus of points equidistant from a fixed point: It is then clearly defined in terms of other mathematical objects. However, for this definition to be meaningful, an individual must see that the locus of points equidistant from a fixed point gives a perfectly round object and vice versa.

We have already referred to mathematical objects as abstract-apart. To emphasise the distinction between abstraction in mathematics and mathematics learning, we shall call empirical concepts abstract-general: Each concept embodies that which is general to the objects from which the similarity is abstracted.

Gravemeier also focuses on how “formal mathematics grows out of the mathematical activity of the students”, calling the process emergent modelling. The Realistic Mathematics Education movement, to which Gravemeier belongs, has previously called it vertical mathematisation. We prefer to call this process formalisation, since its main purpose is to select abstract-apart relationships which capture the form of an abstract-general concept. (So “formal mathematics” is the study of mathematical forms.) For example, the locus definition of a mathematical circle precisely expresses the perfect roundness of an empirical circle.

### **Linking mathematical objects to empirical concepts**

There is strong evidence that many student difficulties in learning mathematics can be traced to the fact that, when they learned about an abstract-apart mathematical object, they made no link to the corresponding abstract-general concept (Mitchelmore & White, 1995). Consider again the previous three examples.

*Addition.* Many young students experience difficulty learning elementary arithmetic. One explanation is that they do not understand the empirical meaning of the are learned apart from the abstract-general%operations: Symbols such as + and concepts of addition and multiplication on which they are based. Early number research (Steffe et al., 1983; Wright, 1994) has led to projects such as Count Me In Too which have closely linked early arithmetic to students’ counting experiences, with a measurable improvement in learning.

*Angle.* Many student difficulties with angles arise because the angle diagram is abstract-apart. Williams (2003) gives a particularly extreme example: Her case-study secondary school student successfully made a generalisation about the angle sum of a polygon, but he could not identify the angles of the triangles into which he had divided the polygon.

*Calculus.* Calculus instruction based on abstract-apart differentiation leads to a manipulation focus (White & Mitchelmore, 1996). Students do not see symbols as representing anything, so they cannot use the manipulative techniques they have learned to solve contextual problems. Their concept of differentiation has been truly decontextualised and therefore impoverished, instead of being abstract-general and rich (Van Oers, 2001)

The preceding discussion emphasises the value of making a clear distinction between empirical concepts and mathematical objects.

### **MORE ADVANCED MATHEMATICS LEARNING**

The learning of fundamental mathematical ideas is only one component of learning mathematics: More advanced ideas need to be developed out of the fundamental ideas. Some of these ideas (such as square roots) can be readily linked back to abstract-general concepts;



others (such as a zero exponent) seem to have no counterpart in normal experience. In addition, students need to learn to operate within an aspect of mathematics learning which takes on an abstract-apart system increasing significance in university mathematics as the links to experience become thinner and thinner. But even professional mathematicians use empirical concepts as an aid to intuition.

The formation of new ideas within mathematics is well described by the SchwarzHershkowitz-Dreyfus Nested RBC Model of Abstraction. They define abstraction as “an activity of vertically reorganizing previously constructed mathematics into a new mathematical structure”. New mathematical objects are constructed by “the establishment of connections, such as inventing a mathematical generalization, proof, or a new strategy of solving a problem”. This abstraction process is quite different from empirical abstraction, and is best described as theoretical abstraction. Sierpinski’s ignoring/highlighting process is another example of theoretical abstraction.

Gray & Tall’s idea of a precept which produces a mathematical object, and a symbol which is used to represent either also clarifies the development of ideas within process or object” mathematics. The construction of a precept seems to us, however, to be more akin to formalisation than abstraction.

Historically, some more advanced mathematical objects have been constructed by a process similar to empirical abstraction. An example is group theory:

The abstract concept of a group arose from different sources. Thus polynomial theory gave rise to groups of permutations, number theory to groups of numbers and of “forms” ... and geometry and analysis to groups of transformations. Common features of these concrete examples of groups began to be noted, and this resulted in the emergence of the abstract concept of a group in the last decades of the 19th century.

Other examples are rings, fields and vector spaces. Our arguments above would suggest that the learning of such mathematics would be most effective if it were based on a process of similarity recognition followed by formalisation.

## **SUMMARY AND CONCLUSION**

The term abstraction has different meanings in relation to mathematics and the learning of mathematics. Previous abstraction theorists have tended to focus on the process of developing ideas within mathematics. In this paper, we have tried to redress the balance by exploring the role of empirical abstraction in the formation of fundamental mathematical ideas. This is a crucial process, since many fundamental, abstract-apart mathematical objects need to be linked to abstract-general empirical concepts if their learning is to be meaningful.

In practice, the formation of mathematics-related empirical concepts and their especially in formalisation into mathematical objects may occur simultaneously school learning. Also, more advanced mathematical objects may be linked directly to empirical concepts and not only indirectly via fundamental objects.

Like Boero, we believe that “we are still far from a comprehensive theoretical answer to the challenge of mathematical abstraction in mathematics education”. A clear response to this challenge would be of great value to researchers and teachers alike. Examining and differentiating the different forms of abstraction involved in learning mathematics constitute one step along the path to this goal.

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# Primitive Pythagorean Triples

December 9, 2019

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## Definition

A *Primitive Pythagorean triple*, or PPT for short, is a triple  $(a, b, c)$  of co-prime positive integers satisfying the relation  $a^2 + b^2 = c^2$ , whereas the Pythagorean triples may not be co-prime (eg.  $(6, 8, 10)$ ). Some well-known PPTs are:  $(3, 4, 5)$ ,  $(5, 12, 13)$  and  $(8, 15, 17)$ .

## Features

Here are some features about PPTs which you need to know in this article (we invite you to provide proofs), If  $(a, b, c)$  is a PPT, then:

1.  $c$  is odd
2. one of out  $a, b$  is odd and other is even.
3. the even number in  $\{a, b\}$  is a multiple of 4.

We agree to list the numbers in the PPT so that  $a$  is the odd number and  $b$  is the even number.

The following property is worth noting:  $b$  is a multiple of 4.

To see why, write  $b^2 = c^2 - a^2$  note that  $a$  and  $c$  are odd, and recall that any odd square is of the form  $(1 \pmod{8})$ . This implies that  $b^2$  is a multiple of 8 and hence that  $b$  is a multiple of 4. (If  $b$  were even but not a multiple of 4, then  $b^2$  would be a multiple of 4 but not a multiple of 8.)

This article focuses on one particular family of PPTs, those having  $b = c - a$ . For this family we have:

$$a^2 + (c - 1)^2 = c^2, \quad a^2 = 2c - 1,$$

so:

$$c = \frac{a^2+1}{2}, \quad b = \frac{a^2-1}{2}$$

This note describes a feature of PPTs  $(a, b, c)$  in which  $b = c - 1$ .

Here are some PPTs with this feature:

$$\begin{aligned} & (3,4,5), & (5,12,13), \\ & (7,24,25), & (9,40,41), \\ & (11,60,61), & (13,84,85), \\ & (15,112,113), & (21,220,221), \\ & (33,544,545), & (35,612,613), \\ & (39,760,761). \end{aligned}$$

**Property:** If  $(a, b, c)$  is a PPT with  $b = c - 1$ , then  $a^b + b^a$  is divisible by  $c$ .

For example:

- For the PPT  $(3,4,5)$ :  $3^4 + 4^3 = 145 = 5 \times 29$ ;
- For the PPT  $(5, 12, 13)$ :  $5^{12} + 12^5 = 244389457 = 13 \times 18799189$

But in the other PPTs such as (15,8,17), (21,20,29), (33,56,65), (35,12,37), (39,80,89), etc., where  $b \neq c - 1$ , this property is not to be seen. Why should the property belong to just this type of PPT?

I will prove the following:

**Theorem:** If  $(a, b, c)$  is a PPT with  $b = c - 1$ , then  $a^b + b^a$  is divisible by  $c$ .

*Proof.*

Since  $b = c - 1$  we have  $c = \frac{a^2+1}{2}$ ,  $b = \frac{a^2-1}{2}$

From  $b = c - 1$ , we get  $b \equiv -1 \pmod{c}$

$b^a \equiv (-1)^a \pmod{c} \equiv -1 \pmod{c}$

Since  $a$  is odd. Next, from  $a^2 = 2c - 1$ , we get  $a^2 \equiv (-1)^a \pmod{c}$ .

Raising both sides to power  $\frac{b}{2}$  (remember that  $b$  is an even number), we get:

$$a^b \equiv (-1)^{\frac{b}{2}} \pmod{c} \equiv 1 \pmod{c}$$

Since,  $b$  is a multiple of 4 (which implies that  $b/2$  is an even number).

Hence

$$a^b + b^a \equiv 1 - 1 \equiv 0 \pmod{c}$$

. In other words  $a^b + b^a$  is divisible by  $c$ .

□

**The End**

**“A STUDY ON EFFECTIVENESS OF GEOGEBRA SOFTWARE AND  
SIMULATIONS ON LEARNING ALGEBRA CONCEPTS IN MATHEMATICS AT  
SECONDARY LEVEL STUDENTS”**

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**ABSTRACT**

The rapid growth of Information and communication technology plays a vital role in the field of education, particularly it is very essential for organizing the teaching and learning process in a meaningful and fruitful way. This Information and communication technology includes the introduction of educational software and other technologies. The use of technology in the pedagogical process is growing at a phenomenal rate due to the vast availability of gadgets. As a result, educationists see the urgent need for integrating technology in students' mathematical activities. Therefore, the purpose of this quasi experimental study was to investigate students' understanding in learning algebra concepts using GeoGebra and simulations regarding algebraic tiling. This study investigates the effectiveness of using Geogebra software and simulations on Mathematics learning among 82 students in Warangal region. Results show that students have positive perception towards learning ( $m = 4.26$ ) and have better learning achievement using Geo Gebra and simulations on algebraic tiling ( $p < 0.05$ ). Available free online, Geo Gebra can benefit students Mathematics learning and diversifying learning algebraic concepts in classrooms. The over flow of free digital resources triggered students' interest to learn Mathematics. However, the selection of software and tools have to be properly planned.

**Key words:** pedagogical process, algebra, geogebra, simulations, algebra tiling, secondary level, mathematics.

## **1. INTRODUCTION:**

Technology has become one of the powerful resources of learning. The evolution in using technology in teaching and learning process has grown by leaps and bounds. There was a lot of Mathematics software have been developed to aid teaching and learning, including GeoGebra.. Several studies have been carried out on GeoGebra software to study various aspects of learning. GeoGebra has become a tool that can help teachers to design effective instructional lessons. GeoGebra not yet widely used in teaching Mathematics in Malaysia. Although, technology has been proven to improve the efficiency of learning. Li (2007) cited that more than 73% of The students commented that GeoGebra is found to be a very useful technology for learning.

Technology allows easy access to information and other cutting-edge research to make learning easier. Teaching and learning with the use of technology has many advantages such as providing greater learning opportunities for students (Roberts, 2012); enhancing student engagement (White, 2012) and encouraging discovery learning (Bennet, 1999). In the teaching and learning of Mathematics, especially geometry, it is important for students to be able to imagine, construct and understand construction of shapes in order to connect them with related facts. Therefore, a computer will assist students in imagining and making observations (Dogan, 2010). A number of technology tools are available such as interactive whiteboards, calculators, Geometers Sketchpad and GeoGebra. This paper will discuss in detail the use of GeoGebra software to conduct learning of circles in mathematics.

### **Statement of Problem**

In the teaching and learning of geometry, it has been often realized that students still lack the cognitive and process abilities in the total understanding of circles. Although the teacher delivers the required knowledge to assist students in understanding the concepts of circles, students seem to face a challenge in applying this knowledge to a given task. It is as though something more is required to guide students so that they are able to manipulate circle properties to truly understand and visualize the properties of circles. This perception is supported by research (Battista,

1999; Prescott, Mitchelmore & White, 2002) whereby students faced challenges in studying geometry and many struggle to grasp the concepts and required knowledge. GeoGebra might play the role in filling up the gap by assisting students to visualize and understand circles through exploration. A review of literature also shows that using GeoGebra has an impact on students' understanding of geometry. Dogan (2010) revealed that GeoGebra had positively affected students' learning and achievement and improved their motivation. Another study by Erhan and Andreasen (2013) also suggested that students improved their mathematics understanding after using the dynamic geometry software. Students were able to explore and form conjectures and therefore had better scores as well. Hence this study entitled **“A STUDY ON EFFECTIVENESS OF GEOGEBRA SOFTWARE AND SIMULATIONS ON LEARNING ALGEBRA CONCEPTS IN MATHEMATICS AT SECONDARY LEVEL STUDENTS”**

### **Objectives and Research Questions:**

The main objective of this study was to investigate the effectiveness of using GeoGebra on students' understanding of circles. Further, the study also aimed at investigating if this learning method surpassed the traditional method and if students perceived learning using technology as useful. The secondary objective was to elicit students' perception in learning circles using GeoGebra.

This study aimed at addressing the following research questions:

1. What is the effectiveness of using GeoGebra on students' understanding of circles as compared to the traditional approach?
2. What were student perceptions about GeoGebra in the learning of circles?

### **HYPOTHESIS:**

- 1) There is a significant difference of the effectiveness of using GeoGebra on students' understanding of circles as compared to the traditional approach
- 2) There are significant students' perceptions about GeoGebra in the learning of circles

### **GeoGebra Software**



GeoGebra was designed by Markus Hohenwarter as an open-source dynamic mathematics software that incorporates geometry, algebra and calculus into a single, open-source, user-friendly package (Hohenwarter, Jarvis, & Lavicza, 2008). This software combined features of older software programs such as Maple, Derive, Cabri and Geometer's Sketchpad (Sahaa, Ayub, & Tarmizi, 2010). GeoGebra is a free and user-friendly software that connects geometry and algebra (White, 2012). GeoGebra's support materials are rather impressive (especially for a free program), where it provides wide-ranging online help feature, 42-page help manual in pdf format, downloadable tutorials, and a variety of detailed lessons using video-based step-by-step examples. These materials are very concise, easily accessible, and professionally done, with supplementary suggestions contributed by users. This concerted assisted environment is described as focusing on "quality versus quantity" in the GeoGebra website (Grandgenett, 2007).

## **METHODOLOGY**

This study applied quasi-experimental research design involving 82 students. Students were divided into two groups of the experimental group and the control group. The experimental group is the group of students was taught about how to use the GeoGebra software to solve geometry concepts and problems. Meanwhile, the control group was given geometry concepts and problems to be solved without using GeoGebra. The participants of the experimental and the control group were randomly selected. The difference between pre and post- performance test determines whether the GeoGebra software influenced the students' achievement on learning Algebra.

### ***Research procedure***

The research procedure consists of four phases. As shown in Figure 1, the first phase is the pre-achievement tests consist of four questions and carried out simultaneously on the experimental group and the control group. The second phase is the intervention phase of the experimental group by using GeoGebra while the control group was taught using traditional teaching methods (without using Geogebra). Next, the third phase is the post-performance test to both groups after two weeks. After the respondents went through the three phases, the test results will be evaluated to determine whether GeoGebra affect student achievement test results for the topic of Algebra

Form 4. At the fourth phase, only the experimental group answered a questionnaire to find out their perception on using GeoGebra

***Samples***

The research sample consisted of Form 2 schools s at a secondary level in Warangal district. There were 82 samples involved where 42 students will be in the control group and another 40 students were in the experimental group. They were chosen based on two classes of form two schools. All students involved have studied the topic circles (Geometry). Therefore, they have a basic knowledge about the subject. *Random sampling technique*

***Instrumentation***

The instrument used in this study is the performance tests; pre-performance test and post-performance test and also a set of questionnaire. The performance tests were used to compare what they knew before in a pre-performance test and what have they experienced in the post-performance test

	<b>Experimental group</b>	<b>Control group</b>
Phase 1	Pre-achievement test	Pre-achievement test
Phase 2	Learning circle topics with GeoGebrasoftware	Learning circle topics without GeoGebrasoftware
Phase-3	Post-achievement test	Post-achievement test

***Performance Tests***

The pre - achievement test was used to determine the achievement level of achievement by students in both groups. This test consists of four questions to be solved without using GeoGebra software that will be answered by both groups experimental and control groups.

Post-performance test contains four questions that have a slightly different with the questions in the pre- performance test, but the question is in the same structure. Post-performance tests used to measure the students’ achievement after using GeoGebra software. These tests involved both the control group and the experimental group. These tests are reliable at  $r = 0.80$  ( $p < 0.05$ ).

***Questionnaires:***

This questionnaire contains nine items using a Likert scale of '1-Strongly Disagree, 2-Disagree, 3-May Agree and May Not Agree, 4-Agree and 5-Strongly Agree'. This study used a modified questionnaire based on Shadaan and Leong (2013) study. This questionnaire contains statements

which reflect the students' perception of the use of GeoGebra software. The questionnaire is reliable with  $\alpha = 0.892$  which indicates good internal consistency.

## RESULTS AND FINDINGS:

### *Students' achievement in learning Mathematics using GeoGebra*

Post test results in Table 1, the difference in the mean scores for the two test results for the two groups of students shows that experimental group performed better than the control group. Based on the findings, it can be stated that the value of  $\alpha = 0.00$  ( $p < 0.05$ ) indicated that the using of GeoGebra software has positive impact on students' achievement in Mathematics. GeoGebra software seems to have a positive effect on the post-performance test conducted on the students. Although the post-performance test scores show a decrease compared to the pre-achievement test, Mann-Whitney U test (Table 1) shows that there are significant differences between the post-performance test on using GeoGebra. As a study conducted by Shadaan and Leong (2013), the results of this study also show that the use of GeoGebra in the learning and teaching process can give a very good impact in improving students' ability.

Table 1. The results of post test ( the Mann-Whitney U test.)

	N	Mean	Standard Deviation	Minimum	Maximum
Post-achievement Test (Control)	42	<b>73.310</b>	15.96168	14.00	100.00
Post-performance Test (Experimental)	40	<b>77.570</b>	17.88072	29.00	100.00
Students	82	1.4839	50.382	1.00	2.00

### *Students' perception towards using GeoGebra for learning algebra*

The students' perception was identified through a set of questionnaire consists of nine items. The questionnaire was distributed to the experimental group only to know their perception based on their experience using the GeoGebra software. The results gained from the questionnaire show positive results. The study found that the items in the questionnaire that had the lowest mean was

the item which stated that students can think creatively and critically with a mean of 3.93. While the highest mean is 4.62, which is obtained for the first item: 'I like using GeoGebra'. Based on Table 2, the overall mean is 4.26. It shows the overall students agreed with positive statements about GeoGebra. Students also found that GeoGebra can also give a good impression of their learning in Mathematics class. From the results, it can be concluded that the using of Geogebra software can increase students' interest, confidence and their motivation in learning Mathematics. Table 2 shows the highest mean of the questionnaire is the first statement: "I like to use GeoGebra software". These students had never used GeoGebra before. Probably these are the reasons why they enjoyed using GeoGebra software in learning Mathematics. Lunar et al., (2010) stated that the use of computers in teaching and learning is not only to improve student performance, but also motivation.

Based on Fig. 2 and Fig. 3, students who responded strongly agree shows the highest percentage compared to other responses. This shows that the students' interest in using GeoGebra software in learning Mathematics.

**Table 2. Table of the mean of questionnaire.**

Item	Minimum	Maximum	Mean	Standard Deviation
1) I like to use GeoGebra software	3	5	4.62	0.561
2) GeoGebra software helps to learn Mathematics concepts	3	5	4.22	0.641
3) I feel confident when do the activities by using GeoGebraSoftware	3	5	4.10	0.618
4) I learnt a lot about Mathematics when using GeoGebra software	2	5	4.21	0.819
5) I can think creatively and critically when using GeoGebra software	2	5	3.9	0.842
6) I prefer to learn Mathematics with GeoGebra software	1	5	4.28	0.996
7) I am excited when asked to				

	explore the GeoGebra software	3	5	4.31	0.761
8)	GeoGebra software can help to				
	increase my achievement in Mathematics	3	5		
		4.25	0.752		
9)	I am happy if the teacher uses the GeoGebra				
	in teaching Mathematics Overall mean	1	5	4.45	0.948

Students may take advantage on the used GeoGebra software in learning Mathematics because they can interact with technology. Students in the 21st century are computer-literate and the opportunities to learn using technology support will attract major attention. They use the Internet, cell phones, computers, laptops, tablets and other software to communicate with others. Digital environment motivates students in the teaching and learning of Mathematics (Korenova, 2012). It also encourage both teachers and students to engage in learning and teaching (Ozdamli, Mus and Nizamoglu, 2013). At present, many scientific studies show that computers have made it easier not only to understand mathematical concepts, but also enhance students' motivation and self-confidence (Yenilmez, 2009).

**Graphs:**

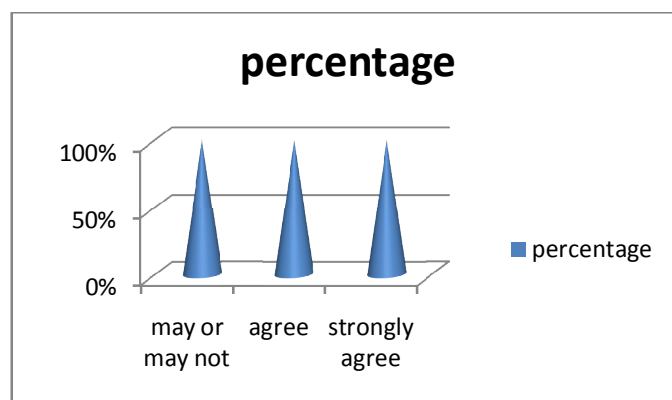
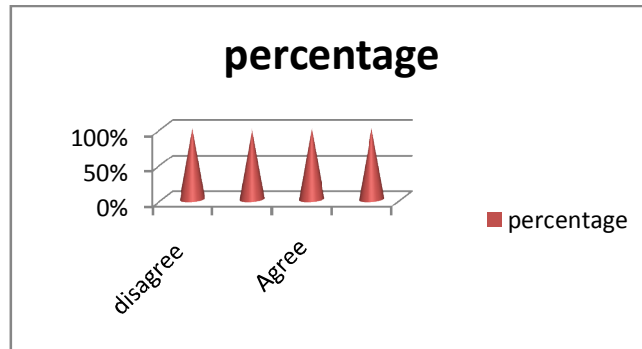
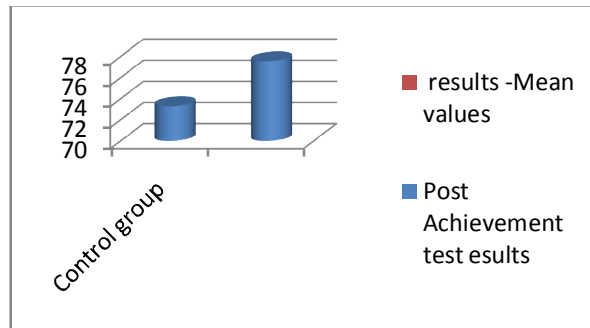


Fig. 2. The graph of the percentage of 'like to use GeoGebra software'.



**Fig. 3. The graph of the percentage of ‘prefer learning Mathematics using GeoGebra software’.**



**Fig.4. Post Achievement test results –mean values**

### **Conclusions:**

In this study, the GeoGebra software has proven to be an effective tool in enhancing Mathematics teaching and learning, specifically in learning circles. Students were able to experience a hands-on method of learning which had a positive effect in enabling them to understand the concepts better rather than just being passive learners.

Learning and teaching of Mathematics should not be focused on purely theoretical, but also a variety of learning approaches that involve the use of teaching aids proven to help stimulate students' interest in Mathematics.

The Mathematics software available in the market or even online has facilitated the task of the teacher to impart knowledge beneficial to the students. However, it depends on the teacher to utilize existing materials without the need to allocate extra time to developed other teaching aids conclusively; this study has shown that GeoGebra software has a positive impact on students' achievement in the topic circles (Geometry). The students also have positive perceptions on GeoGebra software in terms of enthusiasm, confidence, and motivation. This

software should be introduced to Mathematics educators so that students can explore the world of Mathematics in a wider and make the students able to think critically and creatively.

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# Smart Teacher Learning Resources and Tools

Dr Goli. Srilatha\*

## Abstract

Digital tools and Strategies helps the teacher to collaborate, communicate, and demonstrate skills that will ultimately help students navigate their way in the future .Goal of teaching is teacher must have the ability to teach and reach to every student in the classroom. This means the ability to differentiate learning so that all students are able to learn using their own unique style and/or by their ability and readiness level. Innovative strategies and tools in teaching learning support the learner centered education; learners are more active for acquiring the knowledge from different sources all over the globe. So every teacher need to invite innovative methods apart from the traditional methods. Using tools and technology requires not technical training but professional training. Smart teacher role is to explore the user friendly learning resources and ability to foster, real, authentic resources for student's better learning. An effective teacher will have to know how to implement and incorporate technology into the classroom in a way that will be productive for all students. Every teacher require to meet both the 3 R's and the 4 C's(reading ,writing ,arithmetic and creativity, critical thinking, communication ,collaboration) these are possible with the online knowledgeable groups, discussion forums, blogs, OER, Educational Apps ,MOOCs, swayam platform, virtual learning environment (VLE), social media tools and digital recourses. 21<sup>st</sup> century teacher need to learn and use Innovative teaching strategies, digital tools, Professional Development Platforms. In this way, teacher need to be educated and facilitated in students learning according the changing demands and needs of the society.

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## Introduction

Technology supported innovations in education have significant effect on pedagogy of in education. The modern trends in form of Online courses, e learning technologies, social networking tools, and other emerging technologies are being popular among the young generation and further leads to technology supported pedagogical innovation with full access of information communication technology. There is variety of digital tools and technologies available for making pedagogical designs and planning instructional practice for excellent educational experiences. Using tools and technology requires not technical training but professional training. Every teacher should harness the technology and digital tools. These are some digital tools and strategies

## Innovative teaching strategies



**Flipped Classroom Technique:** Flipped classroom is an instructional strategy and a type of blended learning that reverses to the traditional learning environment of the “Chalk and Talk” strategy. It is learner centered strategy in which students explore topics at greater depth creating more opportunities themselves with initial introduction of concepts. Teacher's interaction with student can be more personalized which helps students in knowledge acquisition and self evaluation in learning.

Mathematical concepts, models can be reoriented in this fashion, helping in increasing ease of understanding. It helps in extracting the innovative ideas in students paving way for extracting various types of Models.

Ways of learning through flipped Strategy include video lessons, online discussions, peer group discussion, constructive debates etc.

**Blended Learning:** It is an approach of education that combines online educational material and Opportunities for interaction online with traditional place based classroom methods. It is an approach of introducing Internet based educational system.

The cognitive skills, visual skills and different perspectives of an individual can be increased with the help of Online Videos, Webinars, Power point presentations, Digital Libraries etc.

Blended learning has much to do with Mathematics. Visualization of 2D, 3D Geometry and its understanding is made easy with the help of Presentation, Webinar oriented approach. Many Tools designed in Internet today

**Personalized Learning:** Personalized learning offers an individual with pedagogy, curriculum and course work accordingly. It helps in noticing out the individual areas of interest. It helps in differentiated instruction of support to each individual based on the individual skill mastery.

It also helps in self assessing of their core area of interest, strengths and weakness Online webinars, flipped classroom, Power point presentations, Digital platforms, Interactive Sessions etc can be used in a better way to avoid difficulties in individual learning.

### **Teaching Platforms and Tools**

**Screen sharing:** Online screen sharing methodology is a technique of one to one sharing and enhancing skills simultaneously. This is a personalised system of learning each other.

**Lecture Capture:** Capture of lecture as video or audio files helps in recapitulation of the said Program. It helps for improving the equity of teaching to students. It can be the main feature of Flipped classroom technique. Remote areas with absence of schools colleges can have an edge over it.

**Webinars:** A platform with usage of Internet and Seminar combined together. Interactive sessions and group participation in a task helps us in the knowledge sharing and conceptual clarity. This is a group learning technique.

**File sharing and document management systems:** This methodology of learning is one to many system of file transfer. The file transferred from an individual can be utilized by many. Examples of it include Google drive and Drop box.

**Online Courses:** Online courses are revolutionizing formal education, Before the Digital Age, distance learning appeared in the form of correspondence courses in the 1890s–1920s and later radio and television broadcast of courses and early forms of e-learning. **Massive open online course (MOOCs)** is an online course aimed at unlimited participation and open access via the web. These courses deliver a series of lessons to a web browser or mobile device, to be conveniently accessed by anyone, anytime, anywhere learner able to do it. It's constructed accessed throughout the designated time period by special organizations. It helps in creating and improving Faculty competent environment.

**Electronic Books and Digital resources:** It is also synonymous of E-book. The main features of the E Book is in digital form and exhibits the characteristics of text, images and videos in e-content form. This may be readable on computers and electronic device. Digital resources like mobile learning, web pages, can be used as an alternate way of Learning apart from black board cum class oriented teaching.

### **Assessment Tools**

**Padlet as an assessment Tool:** Padlet is a digital cork board which enables the users to collect and share diverse ideas at a single platform. It looks like an alive webpage. It is called an electronic bulletin board or free virtual wall where documents, text, videos, image, file, web link can be uploaded in form of digital sticky notes by teachers and students. Posts can be written via mobiles, tablets or the computer. Registration is free. This can be encouraging for collaboration and sharing in teaching and learning. For Assessing Students' Knowledge, Understanding of Concepts, knowing Attitudes and Perceptions etc

**Kahoot as an assessment Tool:** Kahoot is known as an assessment tool or feedback device which is used for formative assessment. It is a game centered digital learning platform. It is in fact free student response tool based on gamification. It allows students to evaluate their learned concepts immediately in a very enthusiastic, motivating and encouraging way. Kahoot game can easily be accessed by smart phones or PCs. We can evaluate individually each question based on student responses. It also facilitates surveys and discussions.

**Google Forms:** Create forms with hyperlinks, images, and videos. Use them for surveying and quizzes.

***Poll Everywhere:*** A real-time polling app that works with mobile, Twitter, or in your web browser.

***Socrative.com:*** A free web-based service that lets you assess students with prepared activities or on-the-fly questions to get immediate insight into understanding.

**Nearpod:** Nearpod works in the browser of any device to let you create or upload a slideshow, to which you then add your own questions.

***Classflow:*** Classflow lets you build lessons using cards you create using the content of your choice.

***Classkick:*** Upload a PDF and add text, drawings, photos, hyperlinks, and audio recordings to create dynamic lesson content.

***Seesaw:*** Students show their work with photos, videos, drawings, text, PDFs, and links. You can also import directly from most popular apps.

***Recap:*** A free app that lets teachers prompt the students to explain their thinking on a question or topic using video.

***Quizalize:*** This is a new website for playing class quiz games. The teacher inputs their own questions or they can use a pre-made quiz.

***Triventy:*** This is a tool for making group surveys and quiz games. Again, the teacher can use their own questions or use ones on the site.

***SketchParty TV:*** A Pictionary-style drawing game, perfect for reinforcing vocabulary and visual communication skills.

**Professional Development Platforms:** It is stated that continual and perpetual education is the key to success for further professional development. Professional Development platforms are essential and provide opportunities to explore new knowledge, latest trends and contemporary issues that connect with professional learning communities through technologies. Teachers may get best knowledge through Professional Development platforms and can become popular by having specific training, recognised courses and advanced professional skills to improve and enhance professional knowledge, competence, skill and effectiveness.

- Research gate
- Academia education
- Wiki
- Wikispace
- Swayam platform

## **Conclusion:**

Current system of learning needs a drastic shift in its orientation and strategies of teaching. Practical way of teaching should be the primary agenda unlike the current system of theoretical way of teaching. Technology can be used in creating the best in education like digitalized education, E-Learning, Smart class, App, Tool based education etc.

However how far the technology may take us, education at any period of time either in future or past is nothing without Teacher. The primary source of Education is always a mentor /guide. Technology can be obsolete but Mentor can never be so.

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The Best Quick Formative Assessment Ideas for Teachers to Know

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# Seminar Paper on Mathematics Day

## **Theme:**

Mathematics Teaching for Emerging India

## **SubTheme:**

Mathematics for the Development of Higher Order Thinking Skills

## **Title of the Paper:**

**“The Effect of Teaching Mathematics through Flipped learning strategies for development of higher order thinking skills in 9th grade students of Government Nehru Memorial high school, Malakpet: A quasi Experimental study.”**

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## **The Effect of Teaching Mathematics through Flipped Learning strategies for development of higher order thinking skills in 9th grade students of Government Nehru Memorial high school, Malakpet: A quasi Experimental study.**

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### **ABSTRACT**

This paper investigated the effect of teaching Mathematics through Flipped learning strategies for development of Higher order thinking skills in 9th grade students of Government Nehru memorial high school, Malakpet, Hyderabad. This study used a quasi-experimental method, pretest-posttest nonequivalent groups. It used random sampling to select 30 students as experimental group from 9th grade who were taught using the Flipped Learning approach and 30 students from the same class as control group taught using Traditional method. To conduct the pre-test and post-test paper was prepared by researcher in chapters polynomials, statistics and probability. The pre-test administrated before the implementation of the flipped learning. The instructional strategies that the experimental group received included the following: online instructional videos, guided questions and peer discussions. After 8 weeks the experimental group and control group took post-test. The experimental group and control groups pretest and posttest results were used for the analysis. Mean, standard deviation and t-test were used to compare the two groups performance on the pretest and posttest.

Results show that the students in experimental group had performed better. Findings suggest that teaching Mathematics through flipped learning strategies may be a promising approach to enhance student learning and to develop higher order thinking skills.

Keywords: flipped learning, Higher order thinking skills, 9th grade students

### **1.Introduction:**

Flipped learning (FL) represents a newly emerging form of blended learning, where students individually watch online lectures prior to class and then engage in classroom learning activities interacting with peers and instructors. FL saves time for human instructors to do what only they can do best, and leaves to the technology to do what it can do best. This way of blending posits that neither component has a supplementary role, but rather each is a core complementary constituent of the learning experience as a coherent whole (Stannard 2012).

The term FL derives from Baker's (2000) phrase "the classroom flipped" (p. 9). More recently, Bergmann and Sams (2012) described FL in this way: what is traditionally done in class is done at home, and what is traditionally done as homework is done in class. Learners begin at home, learning with video lectures or screencasts by themselves prior to the class, and then engage in enriching activities that help them apply content at a deeper level (Bergmann and Sams 2013; Collins et al. 2001; Covil et al. 2013; Gannod et al. 2008; Lage et al. 2000; Strayer 2012).

The high technology like computers and calculators has profoundly changed the world of mathematics education. It is not only what aspects of mathematics are essential for learning, but also how mathematics is done and what attitude towards mathematics learning is fostered. Therefore, apart from mathematical content, thinking processes and attitude are also essential core components for mathematics learning at various stages of schooling. Teaching high order thinking skills (HOTS) is currently at the centre of educational attention. In particular, the revised secondary mathematics curriculum has shifted its emphasis to the fostering of HOTS. In general, measures of high order thinking include all intellectual tasks that call for more than the retrieval of information. Therefore, in broad terms, HOTS can be considered as the skills required for performing these tasks. Five fundamental HOTS have been identified in the Syllabus. They are: problem solving skills, reasoning skills, communicating skills, skill of connection and visualization and representation skills.

Problem Solving Skills is an integral part of all mathematics learning and it involves identifying obstacles, constraints or unexpected patterns, trying different procedures and evaluating or justifying the solution. The National Council of Teachers of Mathematics (NCTM) considers problem solving as a process of applying previously acquired knowledge to new and unfamiliar (or unforeseen) situations. To solve a problem, students draw on their knowledge and develop new mathematical understandings. They should also acquire ways of thinking, develop confidence and habits of persistence in unfamiliar situations through the problem solving process.

Reasoning is drawing conclusions from evidence, grounds or assumptions. It involves developing logical arguments to deduce or infer conclusions. Reasoning may be classified into inductive reasoning and deductive reasoning.

Communication involves receiving and sharing ideas and can be expressed in the forms of numbers, symbols, diagrams, graphs, charts, models and simulations. It is viewed as an integral part of mathematics instruction as it helps clarify concepts and build meaning for ideas. Through the communication process, students learn to be clear and convincing in presenting their mathematical ideas, which definitely help develop their logical thinking.

Connecting the concepts in mathematics, connecting math with the concepts of other subjects and real life situations is an essentially a higher order skill for the students to acquire.

Visualizing the abstract nature in mathematics is considered as a higher order thinking skill.

### **Need of the Study**

It is observed that the students who are doing well in their academics are not able to crack competitive exams. Our students are able to qualify NMMS but not NTSE scholarship exams which analyses their higher order thinking skills. There are N.G.Os like Ernst and Young Foundation, Jaladhi Mitra mandali, NTR trust etc. They are conducting Scholarship test to sponsor for higher studies as a part of their CSR. These Scholarship Tests are intended to check the reasoning Skills, problem solving skills, logical skills which are the higher order skills in students.

It is the opinion of most of the maths teachers that the time allotted to them for teaching is not ample to complete the prescribed syllabus. They hardly find time to prepare the students to crack competitive exams. To address this problem, the flipped learning approach is introduced

in the school to optimize the utilization of the class time so that the students get an opportunity to discuss and develop higher order skills under the guidance of the teacher, actively participate in discussions, and develop their critical and analytical thinking abilities. IX class students are selected for conducting this experiment.

### **Background of the study**

Government Nehru Memorial high school is a Telangana state run school that caters to the educational needs of marginalized students from Saidabad habitation. Most of the parents of the school children do not have formal education and work as daily laborers and domestic workers. Our school children do not have computers, laptop or tablets at home. The only source of technological equipment for them at home where they can access the videos and the material posted by the teachers is their fathers' phone.

To bring awareness among the parents to allow their wards to use their phones for flipped learning, a meeting was conducted, and the opinion of the parents has been taken. The parents were convinced that it would benefit their wards and they agreed that their phone could be used when they return from their work in the late evenings. They even said that they keep a watch on their wards, not to get distracted from the purpose for which their phone is being used.

### **2.Objectives of the study:**

1. To compare the mean scores of Pre-Test of Achievement in attaining higher order skills through mathematics of Experimental Group and Control Group.
2. To compare the mean scores of Post-Test of Achievement in attaining higher order skills through mathematics of Experimental Group and Control Group.

### **3. Hypotheses of the study:**

1. There is no significant difference between the mean score of Pre-Test of Achievement in attaining higher order skills through mathematics of Experimental Group and Control Group.
2. There is no significant difference between the mean scores of Post-test of Achievement in attaining higher order skills through mathematics of Experimental Group and Control Group.

### **4.Variables:**

1. Independent variable: Flipped Learning approach.
2. Dependent variable: Achievement in attaining higher order skills through mathematics.

### **5.Methodology:**

Research Design: The selection of appropriate design for this experiment was the essential step in this research. Keeping in view the assorted factors affecting the internal and external validity of the research design, pre-test post-test non-equivalent group design was considered an appropriate research design for this experiment.



### **The Sample:**

60 students of 9th class were divided into two equivalent groups using random sampling technique. In sample, one group was considered as experimental and other as control group. Therefore, the total population for this study was sixty (60).

### **Experimental Procedure:**

To conduct this quasi experimental study, the researcher has chosen the chapters Polynomials, Statistics and Probability to teach for 8 weeks. A pre-test is conducted for both the control group and the experimental group before taking up this experimental study.

A WhatsApp group has been created and 30 students of IX grade were added. A video on the basics of the Chapter Statistics has been prepared initially by the researcher and posted in the WhatsApp group. Online videos on these chapters have been watched by the researcher first and then the relevant part of the video is posted in the groups. When the children are familiar with the terminology and the measures of central tendency in statistics, the questions that require reasoning, analyzing and synthesizing the facts to solve the problems are posed to the children in the class and their discussion was guided by the teacher. The students were made to discuss on the questions under the heading “Think and Discuss” and also the brain teasers from the text book.

The students discussed with their peers about the comparisons to be made for analysis. They participated actively and came out with their observations and conclusions. It was observed that they could understand beyond their IX class text book syllabus.

The videos are recorded by the researchers using a mobile, explaining the concepts on chalk board and the videos are also recorded through screen-o-matic application with the voice over by the researcher while using GeoGebra for making conceptual lessons. The videos made are about 5 to 10 minutes long so that it becomes easy for the students to take time at home to watch. The students even got the opportunity to view the videos repeatedly if they have doubts in understanding the concepts just by watching once. They also discussed about the concept in the group through voice messages. This helped them to improve their communication skills.

For eight weeks, the concepts in the chapters polynomials, Statistics and Probability are introduced through flipped learning approach. The class time, during these eight weeks for the experimental group was utilized for peer discussions, group works and to solve the problems that involves higher order thinking skills.

On the other hand, the control group was taught in the traditional way of introducing the concepts of polynomials, Statistics and Probability in the class and solving the problems and completing the assignments at home. After eight weeks a post test is conducted to both the groups in the chapters taught.

### **Tool for Data Collection Achievement test:**

This study used the achievement test in mathematics developed by the researcher. The validity and reliability analyses of this test were also performed by the researcher.

## Statistical Techniques

To analysis the data following statistical tools were used:

1. Mean score
2. standard deviation and
3. t–test

The analyzed data are presented in below tables.

### **6. Analysis and Interpretation of Data:**

1. There is no significant difference between the mean score of Pre-Test of Achievement in attaining higher order skills through mathematics of Experimental Group and Control Group.

Table 1: Mean score of Pre-Test of achievement in attaining higher order skills through mathematics of Experimental group and Control group

Sl. No.	Group	N	Mean	SD	Df	t-value	Remark
1	Exp	30	95.77	15.38	58	0.64	NS
2	Control	30	93.20	15.71			

The above table shows the mean scores of Pre-Test of Experimental Group and Control Group is 95.77 and 93.20 respectively. The calculated t-value is 0.64, which is not significant at both the level. Hence, the null hypothesis is accepted. Which says that there is no significant difference between the mean score of Pre-Test of achievement in attaining higher order skills through mathematics of Experimental Group and Control Group.

Hypothesis 2: There is no significant difference between the mean scores of Post-Test of Achievement in attaining higher order skills through mathematics of Experimental Group and Control group.

Table 2: Mean score of achievement in attaining higher order skills through mathematics of Post-Test of Experimental Group and Control Group.

Sr. No.	Group	N	Mean	SD	df	t-value	Remark
1	Exp	30	106.27	10.69	58	3.91	S at 0.01
2	Control	30	93.23	14.78			

The table shows the mean scores of Post-Test of Experimental Group and Control Group is 106.27 and 93.23 respectively. The calculated t- value is 3.91, which is significant at 0.01. Hence, the null hypothesis is rejected. Which says that there is no significant difference between the mean score of Post-Test of achievement in attaining higher order skills through mathematics of Experimental Group and Control Group.

### **7. Summary of the findings:**

The students who belong to the experimental group significantly have better achievement scores in attaining higher order skills than those students who belong to the control group. Consequently, flipped learning approach help the students to become active learners and it enhances their level of achievement.

### **8. Conclusion:**

The present study investigated the effect of flipped learning approach on achievement in attaining higher order skills through mathematics of 9th grade students of Government Nehru memorial high school, Malakpet, Hyderabad. The results further indicated that there has been significant positive effect of flipped learning on students in experimental group as compared to students of control group by considering pre-test and post-test scores. Finally, it is concluded that teachers ought to be used these approaches in a school in such a way that each student participates, contributes his maximum and prepares himself for future.

Flipped learning approach seemed to be promising in regards to the benefits it provides for students and teachers. Yet, the research focused on measuring the effectiveness of the new approach through comparisons to traditional approaches, and the conclusions about benefits were reported as additional findings. Therefore, further research is needed to investigate the transferability of these findings to different contexts. Specifically, the claims about the professional skills and increased interaction need to be investigated thoroughly.

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